ภาคผนวก ค.
ตารางปริพันธ์ (Table of Integrals)

รูปแบบมาตรฐาน (Standard Forms)

1. $\int a\,dx = ax + c$

2. $\int af(x)\,dx = a\int f(x)\,dx + C$

3. $\int u\,dv = uv - \int v\,du$ (Integration by parts)

4. $\int u^n\,du = \dfrac{u^{n+1}}{n+1} + C,\ \ n \neq -1$

5. $\int \dfrac{du}{u} = \ln u$ if $u > 0$ or $\ln(-u)$ if $u < 0$
$= \ln|u| + C$

6. $\int e^u\,du = e^u + C$

7. $\int a^u\,du = \int e^{u\ln a}\,du$
$= \dfrac{e^{u\ln a}}{\ln a} = \dfrac{a^u}{\ln a} + C,\ a > 0, a \neq 1$

8. $\int \sin u\,du = -\cos u + C$

9. $\int \cos u\,du = \sin u + C$

10. $\int \tan u\,du = \ln|\sec u| = -\ln|\cos u| + C$

11. $\int \cot u\,du = \ln|\sin u| + C$

12. $\int \sec u\,du = \ln|\sec u + \tan u|$
$= \ln\tan\left|\frac{u}{2} + \frac{\pi}{4}\right| + C$

13. $\int \csc u\,du = \ln|\csc u - \cot u| = \ln\left|\tan\frac{u}{2}\right| + C$

14. $\int \sec^2 u\,du = \tan u + C$

15. $\int \csc^2 u\,du = -\cot u + C$

16. $\int \sec u\tan u\,du = \sec u + C$

17. $\int \csc u\cot u\,du = -\csc u + C$

18. $\int \dfrac{du}{u^2 + a^2} = \dfrac{1}{a}\tan^{-1}\dfrac{u}{a} + C$

19. $\int \dfrac{du}{u^2 - a^2} = \dfrac{1}{2a}\ln\left|\dfrac{u-a}{u+a}\right| + C$
$= -\dfrac{1}{a}\coth^{-1}\dfrac{u}{a} + C,\ u^2 > a^2$

20. $\int \dfrac{du}{a^2 - u^2} = \dfrac{1}{2a}\ln\left|\dfrac{a+u}{a-u}\right| + C$
$= \dfrac{1}{a}\tanh^{-1}\dfrac{u}{a} + C,\ u^2 < a^2$

21. $\int \dfrac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\dfrac{u}{|a|} + C$

22. $\int \dfrac{du}{\sqrt{u^2 + a^2}} = \ln(u + \sqrt{u^2 + a^2}) + C$

23. $\int \dfrac{du}{\sqrt{u^2 - a^2}} = \ln(u + \sqrt{u^2 - a^2}) + C$

24. $\int \dfrac{du}{u\sqrt{u^2 - a^2}} = \dfrac{1}{|a|}\sec^{-1}\left|\dfrac{u}{a}\right| + C$

25. $\int \dfrac{du}{u\sqrt{u^2 + a^2}} = -\dfrac{1}{a}\ln\left|\dfrac{a + \sqrt{u^2 + a^2}}{u}\right| + C$

26. $\int \dfrac{du}{u\sqrt{a^2 - u^2}} = -\dfrac{1}{a}\ln\left|\dfrac{a + \sqrt{a^2 - u^2}}{u}\right| + C$

ปริพันธ์ที่ประกอบด้วยเทอม $au + b$

27. $\int \dfrac{du}{au + b} = \dfrac{1}{a}\ln|au + b| + C$

28. $\int \dfrac{u\,du}{au + b} = \dfrac{u}{a} - \dfrac{b}{a^2}\ln|au + b| + C$

29. $\int \dfrac{u^2\,du}{au + b} = \dfrac{(au+b)^2}{2a^3} - \dfrac{2b(au+b)}{a^3} + \dfrac{b^2}{a^3}\ln|au + b| + C$

30. $\int \frac{du}{u(au+b)} = \frac{1}{b}\ln\left|\frac{u}{au+b}\right| + C$

31. $\int \frac{du}{u^2(au+b)} = -\frac{1}{bu} + \frac{a}{b^2}\ln\left|\frac{au+b}{u}\right| + C$

32. $\int \frac{du}{(au+b)^2} = \frac{-1}{a(au+b)} + C$

33. $\int \frac{udu}{(au+b)^2} = \frac{b}{a^2(au+b)} + \frac{1}{a^2}\ln|au+b| + C$

34. $\int \frac{du}{u(au+b)^2} = \frac{1}{b(au+b)} + \frac{1}{b^2}\ln\left|\frac{u}{au+b}\right| + C$

35. $\int (au+b)^n du = \frac{(au+b)^{n+1}}{(n+1)a} + C,\ n \neq -1$

36. $\int u(au+b)^n du = \frac{(au+b)^{n+2}}{(n+2)a^2} - \frac{b(au+b)^{n+1}}{(n+1)a^2} + C,\ n \neq -1, -2$

37. $\int u^m (au+b)^n du = \begin{cases} \frac{u^{m+1}(au+b)^n}{m+n+1} + \frac{nb}{m+n+1}\int u^m (au+b)^{n-1} du \\ \frac{u^m (au+b)^{n+1}}{(m+n+1)a} + \frac{mb}{(m+n+1)a}\int u^{m-1}(au+b)^n du \\ \frac{-u^{m+1}(au+b)^{n+1}}{(n+1)b} + \frac{m+n+2}{(n+1)b}\int u^m (au+b)^{n+1} du \end{cases}$

ปริพันธ์ที่ประกอบด้วยเทอม $\sqrt{au+b}$

38. $\int \frac{du}{\sqrt{au+b}} = \frac{2\sqrt{au+b}}{a} + C$

39. $\int \frac{udu}{\sqrt{au+b}} = \frac{2(au-2b)}{3a^2}\sqrt{au+b} + C$

40. $\int \frac{du}{u\sqrt{au+b}} = \begin{cases} \frac{1}{\sqrt{b}}\ln\left|\frac{\sqrt{au+b}-\sqrt{b}}{\sqrt{au+b}+\sqrt{b}}\right| + C,\ b > 0 \\ \frac{2}{\sqrt{-b}}\tan^{-1}\sqrt{\frac{au+b}{-b}} + C,\ b < 0 \end{cases}$

41. $\int \sqrt{au+b}du = \frac{2\sqrt{(au+b)^3}}{3a} + C$

42. $\int u\sqrt{au+b}du = \frac{2(3au-2b)}{15a^2}\sqrt{(au+b)^3} + C$

43. $\int \frac{\sqrt{au+b}}{u}du = 2\sqrt{au+b} + b\int \frac{du}{u\sqrt{au+b}} + C$ (ดู ข้อ 40)

ปริพันธ์ที่ประกอบด้วยเทอม u^2+a^2

44. $\int \frac{du}{u^2+a^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$

45. $\int \frac{udu}{u^2+a^2} = \frac{1}{2}\ln(u^2+a^2) + C$

46. $\int \frac{u^2 du}{u^2+a^2} = u - a\tan^{-1}\frac{u}{a} + C$

47. $\int \frac{du}{u(u^2+a^2)} = \frac{1}{2a^2}\ln(\frac{u^2}{u^2+a^2}) + C$

48. $\int \frac{du}{u^2(u^2+a^2)} = -\frac{1}{a^2 u} - \frac{1}{a^3}\tan^{-1}\frac{u}{a} + C$

49. $\int \frac{du}{(u^2+a^2)^n} = \frac{u}{2(n-1)a^2(u^2+a^2)^{n-1}} + \frac{2n-3}{(2n-2)a^2}\int \frac{du}{(u^2+a^2)^{n-1}}$

50. $\int \frac{udu}{(u^2+a^2)^n} = \frac{-1}{2(n-1)(u^2+a^2)^{n-1}} + C$

51. $\int \frac{du}{u(u^2+a^2)^n} = \frac{1}{2(n-1)a^2(u^2+a^2)^{n-1}} + \frac{1}{a^2}\int \frac{du}{u(u^2+a^2)^{n-1}}$

ปริพันธ์ที่ประกอบด้วยเทอม u^2-a^2, $u^2>a^2$

52. $\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$

53. $\int \frac{udu}{u^2-a^2} = \frac{1}{2}\ln(u^2-a^2) + C$

54. $\int \frac{u^2du}{u^2-a^2} = u + \frac{a}{2}\ln\left|\frac{u-a}{u+a}\right| + C$

55. $\int \frac{du}{u(u^2-a^2)} = \frac{1}{2a^2}\ln\left|\frac{u^2-a^2}{u^2}\right| + C$

56. $\int \frac{du}{u^2(u^2-a^2)} = \frac{1}{a^2u} + \frac{1}{2a^3}\ln\left|\frac{u-a}{u+a}\right| + C$

57. $\int \frac{du}{(u^2-a^2)^2} = \frac{-u}{2a^2(u^2-a^2)} - \frac{1}{4a^3}\ln\left|\frac{u-a}{u+a}\right| + C$

58. $\int \frac{du}{(u^2-a^2)^n} = \frac{-u}{2(n-1)a^2(u^2-a^2)^{n-1}} - \frac{2n-3}{(2n-2)a^2}\int \frac{du}{(u^2-a^2)^{n-1}}$

59. $\int \frac{udu}{(u^2-a^2)^n} = \frac{-1}{2(n-1)(u^2-a^2)^{n-1}} + C$

60. $\int \frac{du}{u(u^2-a^2)^n} = \frac{-1}{2(n-1)a^2(u^2-a^2)^{n-1}} - \frac{1}{a^2}\int \frac{du}{u(u^2-a^2)^{n-1}}$

ปริพันธ์ที่ประกอบด้วยเทอม a^2-u^2, $u^2<a^2$

61. $\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{a+u}{a-u}\right| + C$ or $\frac{1}{a}\tanh^{-1}\frac{u}{a} + C$

62. $\int \frac{udu}{a^2-u^2} = -\frac{1}{2}\ln(a^2-u^2) + C$

63. $\int \frac{u^2du}{a^2-u^2} = -u + \frac{a}{2}\ln\left|\frac{a+u}{a-u}\right| + C$

64. $\int \frac{du}{u(u^2-u^2)} = \frac{1}{2a^2}\ln\left|\frac{u^2}{a^2-u^2}\right| + C$

65. $\int \frac{du}{(a^2-u^2)^2} = \frac{u}{2a^2(a^2-u^2)} + \frac{1}{4a^3}\ln\left|\frac{a+u}{a-u}\right| + C$

66. $\int \frac{udu}{(a^2-u^2)^2} = \frac{1}{2(a^2-u^2)} + C$

ปริพันธ์ที่ประกอบด้วยเทอม $\sqrt{u^2+a^2}$

67. $\int \frac{du}{\sqrt{u^2+a^2}} = \ln(u+\sqrt{u^2+a^2}) + C$ or $\sinh^{-1}\frac{u}{|a|} + C$

68. $\int \frac{udu}{\sqrt{u^2+a^2}} = \sqrt{u^2+a^2} + C$

69. $\int \frac{u^2 du}{\sqrt{u^2+a^2}} = \frac{u\sqrt{u^2+a^2}}{2} - \frac{a^2}{2}\ln(u+\sqrt{u^2+a^2}) + C$

70. $\int \frac{du}{u\sqrt{u^2+a^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right| + C$

71. $\int \sqrt{u^2+a^2}\,du = \frac{u\sqrt{u^2+a^2}}{2} + \frac{a^2}{2}\ln(u+\sqrt{u^2+a^2}) + C$

72. $\int u\sqrt{u^2+a^2}\,du = \frac{(u^2+a^2)^{3/2}}{3} + C$

73. $\int u^2\sqrt{u^2+a^2}\,du = \frac{u(u^2+a^2)^{3/2}}{4} - \frac{a^2 u\sqrt{u^2+a^2}}{8} - \frac{a^4}{8}\ln(u+\sqrt{u^2+a^2}) + C$

74. $\int \frac{\sqrt{u^2+a^2}\,du}{u} = \sqrt{u^2+a^2} - a\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right| + C$

75. $\int \frac{\sqrt{u^2+a^2}\,du}{u^2} = -\frac{\sqrt{u^2+a^2}}{u} + \ln(u+\sqrt{u^2+a^2}) + C$

ปริพันธ์ที่ประกอบด้วยเทอม $\sqrt{u^2-a^2}$

76. $\int \frac{du}{\sqrt{u^2-a^2}} = \ln(u+\sqrt{u^2-a^2}) + C$

77. $\int \frac{u\,du}{\sqrt{u^2-a^2}} = \sqrt{u^2-a^2} + C$

78. $\int \frac{u^2 du}{\sqrt{u^2-a^2}} = \frac{u\sqrt{u^2-a^2}}{2} + \frac{a^2}{2}\ln(u+\sqrt{u^2-a^2}) + C$

79. $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C$

80. $\int \sqrt{u^2-a^2}\,du = \frac{u\sqrt{u^2-a^2}}{2} - \frac{a^2}{2}\ln(u+\sqrt{u^2-a^2}) + C$

81. $\int u\sqrt{u^2-a^2}\,du = \frac{(u^2-a^2)^{3/2}}{3} + C$

82. $\int u^2\sqrt{u^2-a^2}\,du = \frac{u(u^2-a^2)^{3/2}}{4} + \frac{a^2 u\sqrt{u^2-a^2}}{8} - \frac{a^4}{8}\ln(u+\sqrt{u^2-a^2}) + C$

83. $\int \frac{\sqrt{u^2-a^2}\,du}{u} = \sqrt{u^2-a^2} - |a|\sec\left|\frac{u}{a}\right| + C$

84. $\int \frac{\sqrt{u^2-a^2}\,du}{u^2} = -\frac{\sqrt{u^2-a^2}}{u} + \ln(u+\sqrt{u^2-a^2}) + C$

85. $\int \frac{du}{(u^2-a^2)^{3/2}} = -\frac{u}{a^2\sqrt{u^2-a^2}} + C$

ปริพันธ์ที่ประกอบด้วยเทอม $\sqrt{a^2-u^2}$

86. $\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + C$

87. $\int \frac{udu}{\sqrt{a^2-u^2}} = -\sqrt{a^2-u^2} + C$

88. $\int \frac{u^2 du}{\sqrt{a^2-u^2}} = -\frac{u\sqrt{a^2-u^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{u}{|a|} + C$

89. $\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$

90. $\int \frac{du}{u^2\sqrt{a^2-u^2}} = -\frac{\sqrt{a^2-u^2}}{a^2 u} + C$

91. $\int \sqrt{a^2-u^2}\,du = \frac{u\sqrt{a^2-u^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{u}{|a|} + C$

92. $\int u\sqrt{a^2-u^2}\,du = -\frac{(a^2-u^2)^{3/2}}{3} + C$

93. $\int u^2\sqrt{a^2-u^2}\,du = -\frac{u(a^2-u^2)^{3/2}}{4} + \frac{a^2 u\sqrt{a^2-u^2}}{8} + \frac{a^4}{8}\sin^{-1}\frac{u}{|a|} + C$

94. $\int \frac{\sqrt{a^2-u^2}}{u}du = \sqrt{a^2-u^2} - a\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$

95. $\int \frac{\sqrt{a^2-u^2}}{u^2}du = -\frac{\sqrt{a^2-u^2}}{u} - \sin^{-1}\frac{u}{|a|} + C$

ปริพันธ์ที่ประกอบด้วยฟังก์ชันตรีโกณมิติ

96. $\int \sin au\,du = -\frac{\cos au}{a} + C$

97. $\int u\sin au\,du = \frac{\sin au}{a^2} - \frac{u\cos au}{a} + C$

98. $\int u^2\sin au\,du = \frac{2u}{a^2}\sin au + \left(\frac{2}{a^3} - \frac{u^2}{a}\right)\cos au + C$

99. $\int \frac{du}{\sin au} = \frac{1}{a}\ln(\csc au - \cot au)$
$= \frac{1}{a}\ln\left|\tan\frac{au}{2}\right| + C$

100. $\int \sin^2 au\,du = \frac{u}{2} - \frac{\sin 2au}{4a} + C$

101. $\int u\sin^2 au\,du = \frac{u^2}{4} - \frac{u\sin 2au}{4a} - \frac{\cos 2au}{8a^2} + C$

102. $\int \frac{du}{\sin^2 au} = -\frac{1}{a}\cot au + C$

103. $\int \sin pu\sin qu\,du = \frac{\sin(p-q)u}{2(p-q)} - \frac{\sin(p+q)u}{2(p+q)} + C,\ p \neq \pm q$

104. $\int \frac{du}{1-\sin au} = \frac{1}{a}\tan\left(\frac{\pi}{4} + \frac{au}{2}\right) + C$

105. $\int \frac{udu}{1-\sin au} = \frac{u}{a}\tan\left(\frac{\pi}{4} + \frac{au}{2}\right) + \frac{2}{a^2}\ln\left|\sin\left(\frac{\pi}{4} - \frac{au}{2}\right)\right| + C$

106. $\int \frac{du}{1+\sin au} = -\frac{1}{a}\tan\left(\frac{\pi}{4} - \frac{au}{2}\right) + C$

107. $$\int \frac{du}{p+q\sin au}=\begin{cases}\dfrac{2}{a\sqrt{p^2-q^2}}\tan^{-1}\dfrac{p\tan\frac{1}{2}au+q}{\sqrt{p^2-q^2}}+C, & |p|>|q| \\ \dfrac{1}{a\sqrt{q^2-p^2}}\ln\left|\dfrac{p\tan\frac{1}{2}au+q-\sqrt{q^2-p^2}}{p\tan\frac{1}{2}au+q+\sqrt{q^2-p^2}}\right|+C, & |p|<|q|\end{cases}$$

108. $$\int u^m \sin au du=-\frac{u^m\cos au}{a}+\frac{mu^{m-1}\sin au}{a^2}-\frac{m(m-1)}{a^2}\int u^{m-2}\sin au du$$

109. $$\int \sin^n au du=-\frac{\sin^{n-1}au\cos au}{an}+\frac{n-1}{n}\int \sin^{n-2}au du$$

110. $$\int \frac{du}{\sin^n au}=\frac{-\cos au}{a(n-1)\sin^{n-1}au}+\frac{n-2}{n-1}\int\frac{du}{\sin^{n-2}au}$$

111. $$\int \cos au du=\frac{\sin au}{a}+C$$

112. $$\int u\cos au du=\frac{\cos au}{a^2}+\frac{u\sin au}{a}+C$$

113. $$\int u^2\cos au du=\frac{2u}{a^2}\cos au+\left(\frac{u^2}{a}-\frac{2}{a^3}\right)\sin au+C$$

114. $$\int \frac{du}{\cos au}=\frac{1}{a}\ln(\sec au+\tan au)=\frac{1}{a}\ln\left|\tan\left(\frac{\pi}{4}+\frac{au}{2}\right)\right|+C$$

115. $$\int \cos^2 au du=\frac{u}{2}+\frac{\sin 2au}{4a}+C$$

116. $$\int \frac{du}{\cos^2 au}=\frac{\tan au}{a}+C$$

117. $$\int u\cos^2 au du=\frac{u^2}{4}+\frac{u\sin 2au}{4a}+\frac{\cos 2au}{8a^2}+C$$

118. $$\int \cos qu\cos pu du=\frac{\sin(q-p)u}{2(q-p)}+\frac{\sin(q+p)u}{2(q+p)}+C,\ q\neq\pm p$$

119. $$\int \frac{du}{p+q\cos au}=\begin{cases}\dfrac{2}{a\sqrt{p^2-q^2}}\tan^{-1}\sqrt{(p-q)/(p+q)}\tan\frac{1}{2}au+C, & |p|>|q| \\ \dfrac{1}{a\sqrt{q^2-p^2}}\ln\left(\dfrac{\tan\frac{1}{2}au+\sqrt{(q+p)/(q-p)}}{\tan\frac{1}{2}au-\sqrt{(q+p)/(q-p)}}\right)+C, & |p|<|q|\end{cases}$$

120. $$\int u^m\cos au du=\frac{u^m\sin au}{a}+\frac{mu^{m-1}\cos au}{a^2}-\frac{m(m-1)}{a^2}\int u^{m-2}\cos au du$$

121. $$\int \cos^n au du=\frac{\sin au\cos^{n-1}au}{an}+\frac{n-1}{n}\int\cos^{n-2}au du$$

122. $$\int \frac{du}{\cos^n au}=\frac{\sin au}{a(n-1)\cos^{n-1}au}+\frac{n-2}{n-1}\int\frac{du}{\cos^{n-2}au}$$

123. $$\int \sin au\cos au du=\frac{\sin^2 au}{2a}+C$$

124. $$\int \sin pu\cos qu du=-\frac{\cos(p-q)u}{2(p-q)}-\frac{\sin(p+q)u}{2(p+q)}+C,\ p\neq\pm q$$

125. $$\int \sin^n au\cos au du=\frac{\sin^{n+1}au}{(n+1)a}+C,\ n\neq -1$$

126. $\int \cos^n au \sin au du = -\frac{\cos^{n+1} au}{(n+1)a} + C,\ n \neq -1$

127. $\int \sin^2 au \cos^2 au du = \frac{u}{8} - \frac{\sin 4au}{32a} + C$

128. $\int \frac{du}{\sin au \cos au} = \frac{1}{a} \ln|\tan au| + C$

129. $\int \frac{du}{\cos au(1 \pm \sin au)} = \mp \frac{1}{2a(1 \pm \sin au)} + \frac{1}{2a} \ln\left|\tan\left(\frac{au}{2} + \frac{\pi}{4}\right)\right| + C$

130. $\int \frac{du}{\sin au(1 \pm \cos au)} = \pm \frac{1}{2a(1 \pm \cos au)} + \frac{1}{2a} \ln\left|\tan \frac{au}{2}\right| + C$

131. $\int \frac{du}{\sin au \pm \cos au} = \frac{1}{a\sqrt{2}} + \frac{1}{2a} \ln\left|\tan\left(\frac{au}{2} \pm \frac{\pi}{8}\right)\right| + C$

132. $\int \frac{\sin au du}{\sin au \pm \cos au} = \frac{u}{2} \mp \frac{1}{2a} \ln|\sin au \pm \cos au| + C$

133. $\int \frac{\cos au du}{\sin au \pm \cos au} = \pm \frac{u}{2} + \frac{1}{2a} \ln|\sin au \pm \cos au| + C$

134. $\int \frac{\sin au du}{p + q\cos au} = -\frac{1}{aq} \ln|p + q\cos au| + C$

135. $\int \frac{\cos au du}{p + q\sin au} = \frac{1}{aq} \ln|p + q\sin au| + C$

136. $\int \sin^m au \cos^n au du = \begin{cases} -\frac{\sin^{m-1} au \cos^{n+1} au}{a(m+n)} + \frac{m-1}{m+n} \int \sin^{m-2} au \cos^n au du \\ \frac{\sin^{m+1} au \cos^{n-1} au}{a(m+n)} + \frac{n-1}{m+n} \int \sin^m au \cos^{n-2} au du \end{cases}$

137. $\int \tan au du = -\frac{1}{a} \ln|\cos au| = \frac{1}{a} \ln|\sec au| + C$

138. $\int \tan^2 au du = \frac{\tan au}{a} - u + C$

139. $\int \tan^n au \sec^2 au du = \frac{\tan^{n+1} au}{(n+1)a} + C$

140. $\int \tan^n au du = \frac{\tan^{n-1} au}{(n-1)a} - \int \tan^{n-2} au du$

141. $\int \cot au du = \frac{1}{a} \ln|\sin au| + C$

142. $\int \cot^2 au du = -\frac{\cot au}{a} - u + C$

143. $\int \cot^n au \csc^2 au du = -\frac{\cot^{n+1} au}{(n+1)a} + C$

144. $\int \cot^n au du = -\frac{\cot^{n-1} au}{(n-1)a} - \int \cot^{n-2} au du$

145. $\int \sec au du = \frac{1}{a} \ln|\sec au + \tan au| = \frac{1}{a} \ln\left|\tan\left(\frac{au}{2} + \frac{\pi}{4}\right)\right| + C$

146. $\int \sec^2 au du = \frac{\tan au}{a} + C$

147. $\int \sec^3 au du = \frac{\sec au \tan au}{2a} + \frac{1}{2a} \ln|\sec au + \tan au| + C$

148. $\int \sec^n au \tan au du = \frac{\sec^n au}{na} + C$

149. $\int \sec^n au du = \frac{\sec^{n-2} au \tan au}{a(n-1)} + \frac{n-2}{n-1} \int \sec^{n-2} au du$

150. $\int \csc au du = \frac{1}{a} \ln|\csc au - \cot au| = \frac{1}{a} \ln\left|\tan \frac{au}{2}\right| + C$

151. $\int \csc^2 au du = -\frac{\cot au}{a} + C$

152. $\int \csc^n au \cot au du = -\frac{\csc^n au}{na} + C$

153. $\int \csc^n au du = \frac{\csc^{n-2} au \cot au}{a(n-1)} + \frac{n-2}{n-1}\int \csc^{n-2} au du$

ปริพันธ์ที่ประกอบด้วยฟังก์ชันตรีโกณมิติผกผัน

154. $\int \sin^{-1}\frac{u}{a} du = u\sin^{-1}\frac{u}{a} + \sqrt{a^2-u^2} + C$

155. $\int u\sin^{-1}\frac{u}{a} du = \left(\frac{u^2}{2} - \frac{a^2}{4}\right)\sin^{-1}\frac{u}{a} + \frac{u\sqrt{a^2-u^2}}{4} + C$

156. $\int \cos^{-1}\frac{u}{a} du = u\cos^{-1}\frac{u}{a} - \sqrt{a^2-u^2} + C$

157. $\int u\cos^{-1}\frac{u}{a} du = \left(\frac{u^2}{2} - \frac{a^2}{4}\right)\cos^{-1}\frac{u}{a} - \frac{u\sqrt{a^2-u^2}}{4} + C$

158. $\int \tan^{-1}\frac{u}{a} du = u\tan^{-1}\frac{u}{a} - \frac{a}{2}\ln(u^2+a^2) + C$

159. $\int u\tan^{-1}\frac{u}{a} du = \frac{1}{2}(u^2+a^2)\tan^{-1}\frac{u}{a} - \frac{au}{2} + C$

160. $\int u^m \sin^{-1}\frac{u}{a} du = \frac{u^{m+1}}{m+1}\sin^{-1}\frac{u}{a} - \frac{1}{m+1}\int \frac{u^{m+1}}{\sqrt{a^2-u^2}} du$

161. $\int u^m \cos^{-1}\frac{u}{a} du = \frac{u^{m+1}}{m+1}\cos^{-1}\frac{u}{a} + \frac{1}{m+1}\int \frac{u^{m+1}}{\sqrt{a^2-u^2}} du$

162. $\int u^m \tan^{-1}\frac{u}{a} du = \frac{u^{m+1}}{m+1}\tan^{-1}\frac{u}{a} - \frac{a}{m+1}\int \frac{u^{m+1}}{\sqrt{a^2-u^2}} du$

ปริพันธ์ที่ประกอบด้วย e^{au}

163. $\int e^{au} du = \frac{e^{au}}{a} + C$

164. $\int ue^{au} du = \frac{e^{au}}{a}\left(u - \frac{1}{a}\right) + C$

165. $\int u^2 e^{au} du = \frac{e^{au}}{a}\left(u^2 - \frac{2u}{a} + \frac{2}{a^2}\right) + C$

166. $\int u^n e^{au} du = \frac{u^n e^{au}}{a} - \frac{n}{a}\int u^{n-1} e^{au} du$

$$= \frac{e^{au}}{a}\left(u^n - \frac{nu^{n-1}}{a} + \frac{n(n-1)u^{n-2}}{a^2} - \ldots \frac{(-1)^n n!}{a^n}\right)$$ if n is a positive integer

167. $\int \frac{du}{p+qe^{au}} = \frac{u}{p} - \frac{1}{ap}\ln\left|p+qe^{au}\right| + C$

168. $\int e^{au}\sin bu du = \frac{e^{au}(a\sin bu - b\cos bu)}{a^2+b^2} + C$

169. $\int e^{au}\cos bu du = \frac{e^{au}(a\cos bu + b\sin bu)}{a^2+b^2} + C$

170. $\int ue^{au}\sin bu du = \frac{ue^{au}(a\sin bu - b\cos bu)}{a^2+b^2} - \frac{e^{au}\left\{(a^2-b^2)\sin bu - 2ab\cos bu\right\}}{(a^2+b^2)^2} + C$

171. $\int ue^{au}\cos bu du = \frac{ue^{au}(a\cos bu + b\sin bu)}{a^2+b^2} - \frac{e^{au}\left\{(a^2-b^2)\cos bu + 2ab\sin bu\right\}}{(a^2+b^2)^2} + C$

172. $\int e^{au}\sin^n bu du = \frac{e^{au}\sin^{n-1} bu}{a^2+n^2b^2}(a\sin bu - nb\cos bu) + \frac{n(n-1)b^2}{a^2+n^2b^2}\int e^{au}\sin^{n-2} bu du$

173. $\int e^{au}\cos^n bu du = \frac{e^{au}\cos^{n-1} bu}{a^2+n^2b^2}(a\cos bu + nb\sin bu) + \frac{n(n-1)b^2}{a^2+n^2b^2}\int e^{au}\cos^{n-2} bu du$

ปริพันธ์ที่ประกอบด้วย ln u

174. $\int \ln u du = u\ln u - u + C$

175. $\int u\ln u du = \frac{u^2}{2}(\ln u - \frac{1}{2}) + C$

176. $\int u^m \ln u du = \frac{u^{m+1}}{m+1}\left(\ln u - \frac{1}{m+1}\right) + C$ if $m \neq -1$

177. $\int \frac{\ln u}{u} du = \frac{1}{2}\ln^2 u + C$

178. $\int \frac{\ln^n u}{u} du = \frac{\ln^{n+1} u}{n+1} + C$ if $n \neq -1$

179. $\int \frac{du}{u\ln u} = \ln|\ln u| + C$

180. $\int \ln^n u du = u\ln^n u - n\int \ln^{n-1} u du + C$

181. $\int u^m \ln^n u du = \frac{u^{m+1}\ln^n u}{m+1} - \frac{n}{m+1}\int u^m \ln^{n-1} u du + C$ if $m \neq -1$

182. $\int \ln(u^2+a^2) du = u\ln(u^2+a^2) - 2u + 2a\tan^{-1}\frac{u}{a} + C$

183. $\int \ln\left|u^2-a^2\right| du = u\ln\left|u^2-a^2\right| - 2u + a\tan^{-1}\left|\frac{u+a}{u-a}\right| + C$

ปริพันธ์ที่ประกอบด้วยฟังก์ชันไฮเพอร์โบลิก

184. $\int \sinh au du = \frac{\cosh au}{a} + C$

185. $\int u\sinh au du = \frac{u\cosh au}{a} - \frac{\sinh au}{a^2} + C$

186. $\int \cosh au du = \frac{\sinh au}{a} + C$

187. $\int u\cosh au du = \frac{u\sinh au}{a} - \frac{\cosh au}{a^2} + C$

188. $\int \cosh^2 au du = \frac{u}{2} + \frac{\sinh au\cosh au}{2a} + C$

189. $\int \sinh^2 au du = \frac{\sinh au\cosh au}{2a} - \frac{u}{2} + C$

190. $\int \sinh^n au du = \frac{\sinh^{n-1} au\cosh au}{an} - \frac{n-1}{n}\int \sinh^{n-2} au du$

191. $\int \cosh^n au du = \frac{\cosh^{n-1} au\sinh au}{an} + \frac{n-1}{n}\int \cosh^{n-2} au du$

192. $\int \sinh au\cosh au du = \frac{\sinh^2 au}{2a} + C$

193. $\int \sinh pu\cosh qu du = \frac{\cosh(p+q)u}{2(p+q)} + \frac{\cosh(p-q)u}{2(p-q)} + C$

194. $\int \tanh au du = \frac{1}{a}\ln\cosh au + C$

195. $\int \tanh^2 au du = u - \frac{\tanh au}{a} + C$

196. $\int \tanh^n au du = \frac{-\tanh^{n-1} au}{a(n-1)} + \int \tanh^{n-2} au du$

197. $\int \coth au du = \frac{1}{a} \ln|\sinh au| + C$

198. $\int \coth^2 au du = u - \frac{\coth au}{a} + C$

199. $\int \operatorname{sech} au du = \frac{2}{a} \tan^{-1} e^{au} + C$

200. $\int \operatorname{sech}^2 au du = \frac{\tanh au}{a} + C$

201. $\int \operatorname{sech}^n au du = \frac{\operatorname{sech}^{n-2} au \tanh au}{a(n-1)} + \frac{n-2}{n-1} \int \operatorname{sech}^{n-2} au du$

202. $\int \operatorname{csch} au du = \frac{1}{a} \ln\left|\tanh \frac{au}{2}\right| + C$

203. $\int \operatorname{csch}^2 au du = -\frac{\coth au}{a} + C$

204. $\int \operatorname{sech} u \tanh u du = -\operatorname{sech} u + C$

205. $\int \operatorname{csch} u \coth u du = -\operatorname{csch} u + C$

ปริพันธ์จำกัดเขตที่สำคัญบางค่า

206. $\int_0^\infty \frac{dx}{x^2 + a^2} = \frac{\pi}{2a}$

207. $\int_0^\infty \frac{x^{p-1}}{1-x} dx = \frac{\pi}{\sin p\pi}$

208. $\int_0^a \frac{dx}{\sqrt{a^2 - x^2}} = \frac{\pi}{2}$

209. $\int_0^a \sqrt{a^2 - x^2}\, dx = \frac{\pi a^2}{4}$

210. $\int_0^\pi \sin mx \sin nx dx = \begin{cases} 0 & \text{if } m, n \text{ integers and } m \neq n \\ \pi/2 & \text{if } m, n \text{ integers and } m = n \end{cases}$

211. $\int_0^\pi \cos mx \cos nx dx = \begin{cases} 0 & \text{if } m, n \text{ integers and } m \neq n \\ \pi/2 & \text{if } m, n \text{ integers and } m = n \end{cases}$

212. $\int_0^\pi \sin mx \cos nx dx = \begin{cases} 0 & \text{if } m, n \text{ integers and } m + n \text{ is odd} \\ 2m/(m^2 - n^2) & \text{if } m, n \text{ integers and } m + n \text{ is even} \end{cases}$

213. $\int_0^{\pi/2} \sin^2 x dx = \int_0^{\pi/2} \cos^2 x dx = \frac{\pi}{4}$

214. $\int_0^\infty e^{-ax} \cos bx dx = \frac{a}{a^2 + b^2}$

215. $\int_0^\infty e^{-ax} \sin bx dx = \frac{b}{a^2 + b^2}$

216. $\int_0^\infty e^{-a^2 x^2} dx = \frac{\sqrt{\pi}}{2a}$

217. $\int_0^{\pi/2} \sin^{2m} x dx = \int_0^{\pi/2} \cos^{2m} x dx = \frac{1 \cdot 3 \cdot 5 \cdots (2m-1)}{2 \cdot 4 \cdot 6 \cdots 2m} \frac{\pi}{2}, \quad m = 1, 2, 3, \ldots$

218. $\int_0^{\pi/2} \sin^{2m+1} x dx = \int_0^{\pi/2} \cos^{2m+1} x dx = \frac{2 \cdot 4 \cdot 6 \cdots 2m}{1 \cdot 3 \cdot 5 \cdots (2m+1)}, \quad m = 1, 2, 3, \ldots$

219. $\int_0^\infty \frac{e^x}{\sqrt{x}} dx = \sqrt{\pi}$

220. $\int_0^1 x^m (\ln x)^n = \frac{(-1)^n n!}{(m+1)^{n+1}}$