

TABLE OF GENERAL PROPERTIES OF LAPLACE TRANSFORMS

$$f(s) = \int_0^{\infty} e^{-st} F(t) dt$$

	$f(s)$	$F(t)$
1.	$a f_1(s) + b f_2(s)$	$a F_1(t) + b F_2(t)$
2.	$f(s/a)$	$a F(at)$
3.	$f(s - a)$	$e^{at} F(t)$
4.	$e^{-as} f(s)$	$u(t-a) = \begin{cases} F(t-a) & t > a \\ 0 & t < a \end{cases}$
5.	$s f(s) - F(0)$	$F'(t)$
6.	$s^2 f(s) - s F(0) - F'(0)$	$F''(t)$
7.	$s^n f(s) - s^{n-1} F(0) - s^{n-2} F'(0) - \dots - F^{(n-1)}(0)$	$F^{(n)}(t)$
8.	$f'(s)$	$-t F(t)$
9.	$f''(s)$	$t^2 F(t)$
10.	$f^{(n)}(s)$	$(-1)^n t^n F(t)$
11.	$\frac{f(s)}{s}$	$\int_0^t F(u) du$
12.	$\frac{f(s)}{s^n}$	$\int_0^t \dots \int_0^t F(u) du^n = \int_0^t \frac{(t-u)^{n-1}}{(n-1)!} F(u) du$
13.	$f(s) g(s)$	$\int_0^t F(u) G(t-u) du$

	$f(s)$	$F(t)$
14.	$\int_s^\infty f(u) du$	$\frac{F(t)}{t}$
15.	$\frac{1}{1 - e^{-sT}} \int_0^T e^{-su} F(u) du$	$F(t) = F(t + T)$
16.	$\frac{f(\sqrt{s})}{s}$	$\frac{1}{\sqrt{\pi t}} \int_0^\infty e^{-u^2/4t} F(u) du$
17.	$\frac{1}{s} f(1/s)$	$\int_0^\infty J_0(2\sqrt{ut}) F(u) du$
18.	$\frac{1}{s^{n+1}} f(1/s)$	$t^{n/2} \int_0^\infty u^{-n/2} J_n(2\sqrt{ut}) F(u) du$
19.	$\frac{f(s + 1/s)}{s^2 + 1}$	$\int_0^t J_0(2\sqrt{u(t-u)}) F(u) du$
20.	$\frac{1}{2\sqrt{\pi}} \int_0^\infty u^{-3/2} e^{-s^2/4u} f(u) du$	$F(t^2)$
21.	$\frac{f(\ln s)}{s \ln s}$	$\int_0^\infty \frac{t^u F(u)}{\Gamma(u+1)} du$
22.	$\frac{P(s)}{Q(s)}$ $P(s) = \text{polynomial of degree less than } n,$ $Q(s) = (s - \alpha_1)(s - \alpha_2) \cdots (s - \alpha_n)$ where $\alpha_1, \alpha_2, \dots, \alpha_n$ are all distinct.	$\sum_{k=1}^n \frac{P(\alpha_k)}{Q'(\alpha_k)} e^{\alpha_k t}$

Appendix B

TABLE OF SPECIAL LAPLACE TRANSFORMS

	$f(s)$	$F(t)$
1.	$\frac{1}{s}$	1
2.	$\frac{1}{s^2}$	t
3.	$\frac{1}{s^n} \quad n = 1, 2, 3, \dots$	$\frac{t^{n-1}}{(n-1)!}, \quad 0! = 1$
4.	$\frac{1}{s^n} \quad n > 0$	$\frac{t^{n-1}}{\Gamma(n)}$
5.	$\frac{1}{s-a}$	e^{at}
6.	$\frac{1}{(s-a)^n} \quad n = 1, 2, 3, \dots$	$\frac{t^{n-1} e^{at}}{(n-1)!}, \quad 0! = 1$
7.	$\frac{1}{(s-a)^n} \quad n > 0$	$\frac{t^{n-1} e^{at}}{\Gamma(n)}$
8.	$\frac{1}{s^2 + a^2}$	$\frac{\sin at}{a}$
9.	$\frac{s}{s^2 + a^2}$	$\cos at$
10.	$\frac{1}{(s-b)^2 + a^2}$	$\frac{e^{bt} \sin at}{a}$
11.	$\frac{s-b}{(s-b)^2 + a^2}$	$e^{bt} \cos at$
12.	$\frac{1}{s^2 - a^2}$	$\frac{\sinh at}{a}$
13.	$\frac{s}{s^2 - a^2}$	$\cosh at$
14.	$\frac{1}{(s-b)^2 - a^2}$	$\frac{e^{bt} \sinh at}{a}$

	$f(s)$	$F(t)$
15.	$\frac{s-b}{(s-b)^2 - a^2}$	$e^{bt} \cosh at$
16.	$\frac{1}{(s-a)(s-b)} \quad a \neq b$	$\frac{e^{bt} - e^{at}}{b-a}$
17.	$\frac{s}{(s-a)(s-b)} \quad a \neq b$	$\frac{be^{bt} - ae^{at}}{b-a}$
18.	$\frac{1}{(s^2 + a^2)^2}$	$\frac{\sin at - at \cos at}{2a^3}$
19.	$\frac{s}{(s^2 + a^2)^2}$	$\frac{t \sin at}{2a}$
20.	$\frac{s^2}{(s^2 + a^2)^2}$	$\frac{\sin at + at \cos at}{2a}$
21.	$\frac{s^3}{(s^2 + a^2)^2}$	$\cos at - \frac{1}{2}at \sin at$
22.	$\frac{s^2 - a^2}{(s^2 + a^2)^2}$	$t \cos at$
23.	$\frac{1}{(s^2 - a^2)^2}$	$\frac{at \cosh at - \sinh at}{2a^3}$
24.	$\frac{s}{(s^2 - a^2)^2}$	$\frac{t \sinh at}{2a}$
25.	$\frac{s^2}{(s^2 - a^2)^2}$	$\frac{\sinh at + at \cosh at}{2a}$
26.	$\frac{s^3}{(s^2 - a^2)^2}$	$\cosh at + \frac{1}{2}at \sinh at$
27.	$\frac{s^2 + a^2}{(s^2 - a^2)^2}$	$t \cosh at$
28.	$\frac{1}{(s^2 + a^2)^3}$	$\frac{(3 - a^2t^2) \sin at - 3at \cos at}{8a^5}$
29.	$\frac{s}{(s^2 + a^2)^3}$	$\frac{t \sin at - at^2 \cos at}{8a^3}$
30.	$\frac{s^2}{(s^2 + a^2)^3}$	$\frac{(1 + a^2t^2) \sin at - at \cos at}{8a^3}$
31.	$\frac{s^3}{(s^2 + a^2)^3}$	$\frac{3t \sin at + at^2 \cos at}{8a}$

	$f(s)$	$F(t)$
32.	$\frac{s^4}{(s^2 + a^2)^3}$	$\frac{(3 - a^2 t^2) \sin at + 5at \cos at}{8a}$
33.	$\frac{s^5}{(s^2 + a^2)^3}$	$\frac{(8 - a^2 t^2) \cos at - 7at \sin at}{8}$
34.	$\frac{3s^2 - a^2}{(s^2 + a^2)^3}$	$\frac{t^2 \sin at}{2a}$
35.	$\frac{s^3 - 3a^2 s}{(s^2 + a^2)^3}$	$\frac{1}{2} t^2 \cos at$
36.	$\frac{s^4 - 6a^2 s^2 + a^4}{(s^2 + a^2)^4}$	$\frac{1}{6} t^3 \cos at$
37.	$\frac{s^3 - a^2 s}{(s^2 + a^2)^4}$	$\frac{t^3 \sin at}{24a}$
38.	$\frac{1}{(s^2 - a^2)^3}$	$\frac{(3 + a^2 t^2) \sinh at - 3at \cosh at}{8a^5}$
39.	$\frac{s}{(s^2 - a^2)^3}$	$\frac{at^2 \cosh at - t \sinh at}{8a^3}$
40.	$\frac{s^2}{(s^2 - a^2)^3}$	$\frac{at \cosh at + (a^2 t^2 - 1) \sinh at}{8a^3}$
41.	$\frac{s^3}{(s^2 - a^2)^3}$	$\frac{3t \sinh at + at^2 \cosh at}{8a}$
42.	$\frac{s^4}{(s^2 - a^2)^3}$	$\frac{(3 + a^2 t^2) \sinh at + 5at \cosh at}{8a}$
43.	$\frac{s^5}{(s^2 - a^2)^3}$	$\frac{(8 + a^2 t^2) \cosh at + 7at \sinh at}{8}$
44.	$\frac{3s^2 + a^2}{(s^2 - a^2)^3}$	$\frac{t^2 \sinh at}{2a}$
45.	$\frac{s^3 + 3a^2 s}{(s^2 - a^2)^3}$	$\frac{1}{2} t^2 \cosh at$
46.	$\frac{s^4 + 6a^2 s^2 + a^4}{(s^2 - a^2)^4}$	$\frac{1}{6} t^3 \cosh at$
47.	$\frac{s^3 + a^2 s}{(s^2 - a^2)^4}$	$\frac{t^3 \sinh at}{24a}$
48.	$\frac{1}{s^3 + a^3}$	$\frac{eat/2}{3a^2} \left\{ \sqrt{3} \sin \frac{\sqrt{3} at}{2} - \cos \frac{\sqrt{3} at}{2} + e^{-3at/2} \right\}$

	$f(s)$	$F(t)$
49.	$\frac{s}{s^3 + a^3}$	$\frac{e^{at/2}}{3a} \left\{ \cos \frac{\sqrt{3} at}{2} + \sqrt{3} \sin \frac{\sqrt{3} at}{2} - e^{-3at/2} \right\}$
50.	$\frac{s^2}{s^3 + a^3}$	$\frac{1}{3} \left(e^{-at} + 2e^{at/2} \cos \frac{\sqrt{3} at}{2} \right)$
51.	$\frac{1}{s^3 - a^3}$	$\frac{e^{-at/2}}{3a^2} \left\{ e^{3at/2} - \cos \frac{\sqrt{3} at}{2} - \sqrt{3} \sin \frac{\sqrt{3} at}{2} \right\}$
52.	$\frac{s}{s^3 - a^3}$	$\frac{e^{-at/2}}{3a} \left\{ \sqrt{3} \sin \frac{\sqrt{3} at}{2} - \cos \frac{\sqrt{3} at}{2} + e^{3at/2} \right\}$
53.	$\frac{s^2}{s^3 - a^3}$	$\frac{1}{3} \left(e^{at} + 2e^{-at/2} \cos \frac{\sqrt{3} at}{2} \right)$
54.	$\frac{1}{s^4 + 4a^4}$	$\frac{1}{4a^3} (\sin at \cosh at - \cos at \sinh at)$
55.	$\frac{s}{s^4 + 4a^4}$	$\frac{\sin at \sinh at}{2a^2}$
56.	$\frac{s^2}{s^4 + 4a^4}$	$\frac{1}{2a} (\sin at \cosh at + \cos at \sinh at)$
57.	$\frac{s^3}{s^4 + 4a^4}$	$\cos at \cosh at$
58.	$\frac{1}{s^4 - a^4}$	$\frac{1}{2a^3} (\sinh at - \sin at)$
59.	$\frac{s}{s^4 - a^4}$	$\frac{1}{2a^2} (\cosh at - \cos at)$
60.	$\frac{s^2}{s^4 - a^4}$	$\frac{1}{2a} (\sinh at + \sin at)$
61.	$\frac{s^3}{s^4 - a^4}$	$\frac{1}{2} (\cosh at + \cos at)$
62.	$\frac{1}{\sqrt{s+a} + \sqrt{s+b}}$	$\frac{e^{-bt} - e^{-at}}{2(b-a)\sqrt{\pi t^3}}$
63.	$\frac{1}{s\sqrt{s+a}}$	$\frac{\operatorname{erf} \sqrt{at}}{\sqrt{a}}$
64.	$\frac{1}{\sqrt{s}(s-a)}$	$\frac{e^{at} \operatorname{erf} \sqrt{at}}{\sqrt{a}}$
65.	$\frac{1}{\sqrt{s-a} + b}$	$e^{at} \left\{ \frac{1}{\sqrt{\pi t}} - b e^{b^2 t} \operatorname{erfc}(b\sqrt{t}) \right\}$

	$f(s)$	$F(t)$
66.	$\frac{1}{\sqrt{s^2 + a^2}}$	$J_0(at)$
67.	$\frac{1}{\sqrt{s^2 - a^2}}$	$I_0(at)$
68.	$\frac{(\sqrt{s^2 + a^2} - s)^n}{\sqrt{s^2 + a^2}} \quad n > -1$	$a^n J_n(at)$
69.	$\frac{(s - \sqrt{s^2 - a^2})^n}{\sqrt{s^2 - a^2}} \quad n > -1$	$a^n I_n(at)$
70.	$\frac{e^{b(s - \sqrt{s^2 + a^2})}}{\sqrt{s^2 + a^2}}$	$J_0(a\sqrt{t(t+2b)})$
71.	$\frac{e^{-b\sqrt{s^2 + a^2}}}{\sqrt{s^2 + a^2}}$	$\begin{cases} J_0(a\sqrt{t^2 - b^2}) & t > b \\ 0 & t < b \end{cases}$
72.	$\frac{1}{(s^2 + a^2)^{3/2}}$	$\frac{t J_1(at)}{a}$
73.	$\frac{s}{(s^2 + a^2)^{3/2}}$	$t J_0(at)$
74.	$\frac{s^2}{(s^2 + a^2)^{3/2}}$	$J_0(at) - at J_1(at)$
75.	$\frac{1}{(s^2 - a^2)^{3/2}}$	$\frac{t I_1(at)}{a}$
76.	$\frac{s}{(s^2 - a^2)^{3/2}}$	$t I_0(at)$
77.	$\frac{s^2}{(s^2 - a^2)^{3/2}}$	$I_0(at) + at I_1(at)$
78.	$\frac{1}{s(e^s - 1)} = \frac{e^{-s}}{s(1 - e^{-s})}$ See also entry 141, Page 254.	$F(t) = n, \quad n \leq t < n + 1, \quad n = 0, 1, 2, \dots$
79.	$\frac{1}{s(e^s - r)} = \frac{e^{-s}}{s(1 - re^{-s})}$	$F(t) = \sum_{k=1}^{[t]} r^k$ where $[t]$ = greatest integer $\leq t$
80.	$\frac{e^s - 1}{s(e^s - r)} = \frac{1 - e^{-s}}{s(1 - re^{-s})}$ See also entry 143, Page 254.	$F(t) = r^n, \quad n \leq t < n + 1, \quad n = 0, 1, 2, \dots$
81.	$\frac{e^{-a/s}}{\sqrt{s}}$	$\frac{\cos 2\sqrt{at}}{\sqrt{\pi t}}$

	$f(s)$	$F(t)$
82.	$\frac{e^{-a/s}}{s^{3/2}}$	$\frac{\sin 2\sqrt{at}}{\sqrt{\pi a}}$
83.	$\frac{e^{-a/s}}{s^{n+1}} \quad n > -1$	$\left(\frac{t}{a}\right)^{n/2} J_n(2\sqrt{at})$
84.	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}}$	$\frac{e^{-a^2/4t}}{\sqrt{\pi t}}$
85.	$e^{-a\sqrt{s}}$	$\frac{a}{2\sqrt{\pi t^3}} e^{-a^2/4t}$
86.	$\frac{1 - e^{-a\sqrt{s}}}{s}$	$\operatorname{erf}(a/2\sqrt{t})$
87.	$\frac{e^{-a\sqrt{s}}}{s}$	$\operatorname{erfc}(a/2\sqrt{t})$
88.	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}(\sqrt{s}+b)}$	$e^{b(bt+a)} \operatorname{erfc}\left(b\sqrt{t} + \frac{a}{2\sqrt{t}}\right)$
89.	$\frac{e^{-a/\sqrt{s}}}{s^{n+1}} \quad n > -1$	$\frac{1}{\sqrt{\pi t} a^{2n+1}} \int_0^\infty u^n e^{-u^2/4a^2t} J_{2n}(2\sqrt{u}) du$
90.	$\ln\left(\frac{s+a}{s+b}\right)$	$\frac{e^{-bt} - e^{-at}}{t}$
91.	$\frac{\ln[(s^2+a^2)/a^2]}{2s}$	$\operatorname{Ci}(at)$
92.	$\frac{\ln[(s+a)/a]}{s}$	$\operatorname{Ei}(at)$
93.	$-\frac{(\gamma + \ln s)}{s}$ $\gamma = \text{Euler's constant} = .5772156\dots$	$\ln t$
94.	$\ln\left(\frac{s^2+a^2}{s^2+b^2}\right)$	$\frac{2(\cos at - \cos bt)}{t}$
95.	$\frac{\pi^2}{6s} + \frac{(\gamma + \ln s)^2}{s}$ $\gamma = \text{Euler's constant} = .5772156\dots$	$\ln^2 t$
96.	$\frac{\ln s}{s}$	$-(\ln t + \gamma)$ $\gamma = \text{Euler's constant} = .5772156\dots$
97.	$\frac{\ln^2 s}{s}$	$(\ln t + \gamma)^2 - \frac{1}{6}\pi^2$ $\gamma = \text{Euler's constant} = .5772156\dots$

	$f(s)$	$F(t)$
98.	$\frac{\Gamma'(n+1) - \Gamma(n+1) \ln s}{s^{n+1}} \quad n > -1$	$t^n \ln t$
99.	$\tan^{-1}(a/s)$	$\frac{\sin at}{t}$
100.	$\frac{\tan^{-1}(a/s)}{s}$	$\text{Si}(at)$
101.	$\frac{e^{a/s}}{\sqrt{s}} \text{erfc}(\sqrt{a/s})$	$\frac{e^{-2\sqrt{at}}}{\sqrt{\pi t}}$
102.	$e^{s^2/4a^2} \text{erfc}(s/2a)$	$\frac{2a}{\sqrt{\pi}} e^{-a^2 t^2}$
103.	$\frac{e^{s^2/4a^2} \text{erfc}(s/2a)}{s}$	$\text{erf}(at)$
104.	$\frac{e^{as} \text{erfc} \sqrt{as}}{\sqrt{s}}$	$\frac{1}{\sqrt{\pi(t+a)}}$
105.	$e^{as} \text{Ei}(as)$	$\frac{1}{t+a}$
106.	$\frac{1}{a} \left[\cos as \left\{ \frac{\pi}{2} - \text{Si}(as) \right\} - \sin as \text{Ci}(as) \right]$	$\frac{1}{t^2 + a^2}$
107.	$\sin as \left\{ \frac{\pi}{2} - \text{Si}(as) \right\} + \cos as \text{Ci}(as)$	$\frac{t}{t^2 + a^2}$
108.	$\frac{\cos as \left\{ \frac{\pi}{2} - \text{Si}(as) \right\} - \sin as \text{Ci}(as)}{s}$	$\tan^{-1}(t/a)$
109.	$\frac{\sin as \left\{ \frac{\pi}{2} - \text{Si}(as) \right\} + \cos as \text{Ci}(as)}{s}$	$\frac{1}{2} \ln \left(\frac{t^2 + a^2}{a^2} \right)$
110.	$\left[\frac{\pi}{2} - \text{Si}(as) \right]^2 + \text{Ci}^2(as)$	$\frac{1}{t} \ln \left(\frac{t^2 + a^2}{a^2} \right)$
111.	0	$\mathcal{N}(t)$
112.	1	$\delta(t)$
113.	e^{-as}	$\delta(t-a)$
114.	$\frac{e^{-as}}{s}$	$u(t-a)$

See also entry 139, Page 254.

	$f(s)$	$F(t)$
115.	$\frac{\sinh sx}{s \sinh sa}$	$\frac{x}{a} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi x}{a} \cos \frac{n\pi t}{a}$
116.	$\frac{\sinh sx}{s \cosh sa}$	$\frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1} \sin \frac{(2n-1)\pi x}{2a} \sin \frac{(2n-1)\pi t}{2a}$
117.	$\frac{\cosh sx}{s \sinh sa}$	$\frac{t}{a} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \cos \frac{n\pi x}{a} \sin \frac{n\pi t}{a}$
118.	$\frac{\cosh sx}{s \cosh sa}$	$1 + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1} \cos \frac{(2n-1)\pi x}{2a} \cos \frac{(2n-1)\pi t}{2a}$
119.	$\frac{\sinh sx}{s^2 \sinh sa}$	$\frac{xt}{a} + \frac{2a}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \sin \frac{n\pi x}{a} \sin \frac{n\pi t}{a}$
120.	$\frac{\sinh sx}{s^2 \cosh sa}$	$x + \frac{8a}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)^2} \sin \frac{(2n-1)\pi x}{2a} \cos \frac{(2n-1)\pi t}{2a}$
121.	$\frac{\cosh sx}{s^2 \sinh sa}$	$\frac{t^2}{2a} + \frac{2a}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos \frac{n\pi x}{a} \left(1 - \cos \frac{n\pi t}{a}\right)$
122.	$\frac{\cosh sx}{s^2 \cosh sa}$	$t + \frac{8a}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)^2} \cos \frac{(2n-1)\pi x}{2a} \sin \frac{(2n-1)\pi t}{2a}$
123.	$\frac{\cosh sx}{s^3 \cosh sa}$	$\frac{1}{2}(t^2 + x^2 - a^2) - \frac{16a^2}{\pi^3} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)^3} \cos \frac{(2n-1)\pi x}{2a} \cos \frac{(2n-1)\pi t}{2a}$
124.	$\frac{\sinh x\sqrt{s}}{\sinh a\sqrt{s}}$	$\frac{2\pi}{a^2} \sum_{n=1}^{\infty} (-1)^n n e^{-n^2\pi^2 t/a^2} \sin \frac{n\pi x}{a}$
125.	$\frac{\cosh x\sqrt{s}}{\cosh a\sqrt{s}}$	$\frac{\pi}{a^2} \sum_{n=1}^{\infty} (-1)^{n-1} (2n-1) e^{-(2n-1)^2\pi^2 t/4a^2} \cos \frac{(2n-1)\pi x}{2a}$
126.	$\frac{\sinh x\sqrt{s}}{\sqrt{s} \cosh a\sqrt{s}}$	$\frac{2}{a} \sum_{n=1}^{\infty} (-1)^{n-1} e^{-(2n-1)^2\pi^2 t/4a^2} \sin \frac{(2n-1)\pi x}{2a}$
127.	$\frac{\cosh x\sqrt{s}}{\sqrt{s} \sinh a\sqrt{s}}$	$\frac{1}{a} + \frac{2}{a} \sum_{n=1}^{\infty} (-1)^n e^{-n^2\pi^2 t/a^2} \cos \frac{n\pi x}{a}$
128.	$\frac{\sinh x\sqrt{s}}{s \sinh a\sqrt{s}}$	$\frac{x}{a} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} e^{-n^2\pi^2 t/a^2} \sin \frac{n\pi x}{a}$
129.	$\frac{\cosh x\sqrt{s}}{s \cosh a\sqrt{s}}$	$1 + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1} e^{-(2n-1)^2\pi^2 t/4a^2} \cos \frac{(2n-1)\pi x}{2a}$
130.	$\frac{\sinh x\sqrt{s}}{s^2 \sinh a\sqrt{s}}$	$\frac{xt}{a} + \frac{2a^2}{\pi^3} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} (1 - e^{-n^2\pi^2 t/a^2}) \sin \frac{n\pi x}{a}$
131.	$\frac{\cosh x\sqrt{s}}{s^2 \cosh a\sqrt{s}}$	$\frac{1}{2}(x^2 - a^2) + t - \frac{16a^2}{\pi^3} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)^3} e^{-(2n-1)^2\pi^2 t/4a^2} \cos \frac{(2n-1)\pi x}{2a}$