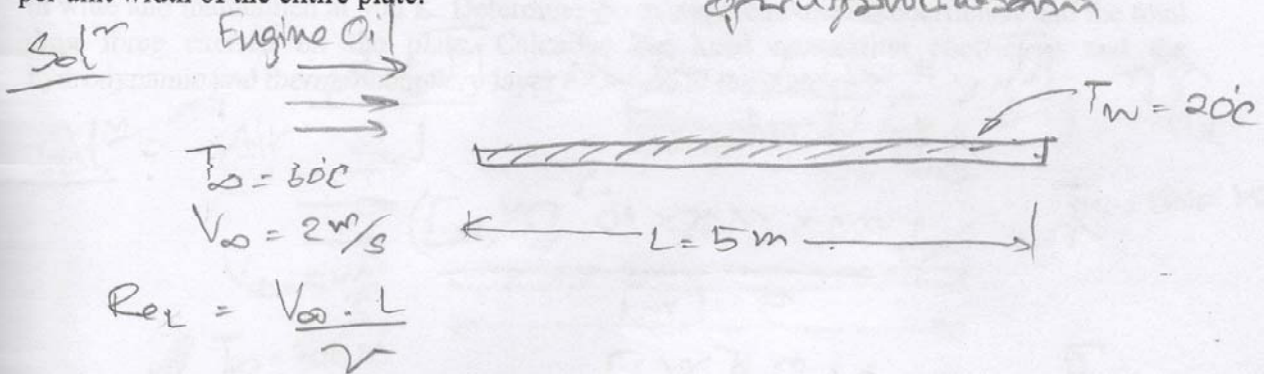


Example 6.1

Engine oil at 60°C flows over the upper surface of a 5 m long flat plate whose temperature is 20°C with a velocity of 2 m/sec. Determine the total drag force and the rate of heat transfer per unit width of the entire plate.



$$Re_L = \frac{V_\infty \cdot L}{\nu}$$

for Engine Oil; properties at $T_{av} = \frac{T_\infty + T_w}{2} = \frac{60 + 20}{2} = 40^\circ\text{C}$
 at $40^\circ\text{C} \rightarrow \nu = 288 \times 10^{-6} \frac{\text{m}^2}{\text{s}}$, $Pr = 3400$, $\rho = 877.9 \frac{\text{kg}}{\text{m}^3}$, $k_f = 145 \times 10^{-3} \frac{\text{W}}{\text{m}\cdot^\circ\text{C}}$

$$\therefore Re_L = \frac{2 \left[\frac{\text{m}}{\text{s}} \right] \cdot 5 \text{ [m]}}{288 \times 10^{-6} \left[\frac{\text{m}^2}{\text{s}} \right]} = 0.347 \times 10^5 < Re_{crit} = 5 \times 10^5$$

\therefore flow is laminar over the entire length of 5 m.

For laminar flow

$$\textcircled{1} \text{ Drag force; } D_f = \frac{1}{2} C_D \rho V_\infty^2 A$$

$$\text{So } C_D = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{0.347 \times 10^5}} = 0.007129$$

$$\therefore D_f = \frac{1}{2} (0.007129) \cdot (877.9 \left[\frac{\text{kg}}{\text{m}^3} \right]) \cdot (2 \left[\frac{\text{m}}{\text{s}} \right])^2 \cdot (5 \text{ [m]}) \text{ [m]} = 62.59 \text{ [N]}$$

Rate of heat transfer per unit width of the plate $\left(\frac{D_f}{W} \right)$

$$\begin{aligned} \frac{D_f}{W} &= \frac{1}{2} (0.007129) \cdot (877.9 \left[\frac{\text{kg}}{\text{m}^3} \right]) \cdot (2 \left[\frac{\text{m}}{\text{s}} \right])^2 \cdot (5 \text{ [m]}) \\ &= 62.59 \left[\frac{\text{kg}}{\text{s}^2} \right] \Rightarrow = 62.59 \left[\frac{\text{N}}{\text{m}} \right] \end{aligned}$$

$\textcircled{2}$ Rate of heat transfer

$$Q = \bar{h} \cdot A \cdot (T_\infty - T_w) = \bar{h} \cdot (W \cdot L) \cdot (T_\infty - T_w)$$

$$\Rightarrow \text{Rate of heat transfer per unit width of the plate} = \bar{h} \cdot L \cdot (T_\infty - T_w)$$

$$\frac{Q}{W} = \bar{h} \cdot L \cdot (T_\infty - T_w)$$

სიმართლისთვის, მისივე გამოყენებით,

$$\overline{Nu}_L = 0.644 Re_L^{\frac{1}{2}} Pr^{\frac{1}{3}}$$

$$\overline{Nu}_L = 0.644 (0.347 \times 10^5)^{\frac{1}{2}} (3400)^{\frac{1}{3}} = 1,804$$

ან $\overline{Nu}_L = \frac{\overline{h} \cdot L}{k_f} \rightarrow \overline{h} = \frac{\overline{Nu}_L \cdot k_f}{L}$

$$\overline{h} = \frac{1,804 \times (145 \times 10^{-3} \text{ [W/m} \cdot \text{K]})}{5 \text{ [m]}}$$

$$\therefore \overline{h} = 52.31 \text{ [W/m}^2 \cdot \text{K]}$$

$$\therefore \frac{Q}{W} = 52.31 \text{ [W/m}^2 \cdot \text{K]} \cdot 5 \text{ [m]} \cdot (60 - 20) \text{ [K]}$$

$$\therefore \frac{Q}{2} = 10,463 \text{ [W/m]}$$

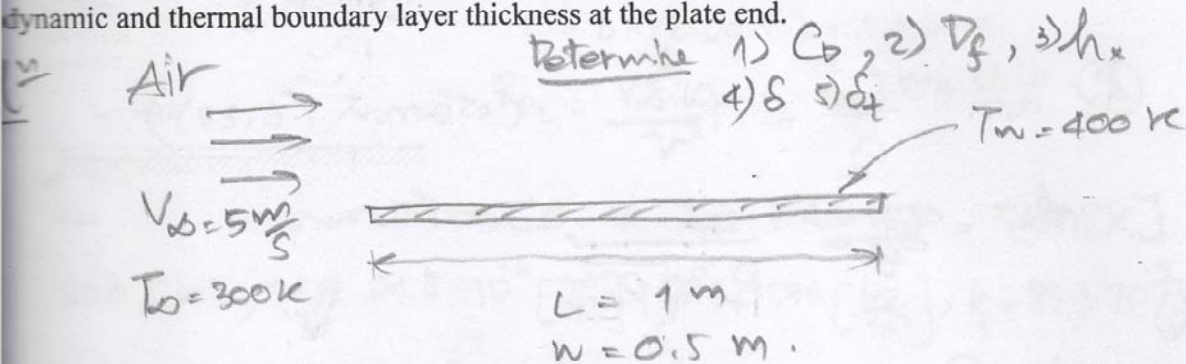
ბოლოს

- სითბოს გამართვის მაჩვენებელი = 62.59 [N/m]

- სითბოს გამართვის მაჩვენებელი = $10,463 \text{ [W/m]}$

Example 6.2

Air at 300 K moves at 5 m/sec past a warmed isothermal flat plate. The plate is 1 m long, 0.5 m wide and maintained at 400 K. Determine the average convection coefficient and the total force exerted on the plate. Calculate the local convection coefficient and the dynamic and thermal boundary layer thickness at the plate end.



$$Re_L = \frac{V_\infty \cdot L}{\nu}$$

Average Temperature $T_{av} = \frac{T_\infty + T_w}{2} = \frac{300 + 400}{2} = 350 \text{ (K)}$

350 (K) ; $\nu = 20.92 \times 10^{-6} \left[\frac{\text{m}^2}{\text{s}} \right]$, $\rho = 0.9950 \left[\frac{\text{kg}}{\text{m}^3} \right]$, $k_f = 30 \times 10^{-3} \left[\frac{\text{W}}{\text{m} \cdot \text{K}} \right]$

$Pr = 0.7$

$$Re_L = \frac{5 \left[\frac{\text{m}}{\text{s}} \right] \cdot 1 \text{ (m)}}{20.92 \times 10^{-6} \left[\frac{\text{m}^2}{\text{s}} \right]} = 2.39 \times 10^5 < Re_{crit}$$

Transition to Turbulence

Laminar Flow:

1) C_D ; $C_D = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{2.39 \times 10^5}} = 0.002716$ — Ans

2) $D_f = \frac{1}{2} C_D \rho V_\infty^2 \cdot A = \frac{1}{2} (0.002716) (0.995 \left[\frac{\text{kg}}{\text{m}^3} \right]) (5 \left[\frac{\text{m}}{\text{s}} \right])^2 (1 \times 0.5 \text{ (m)}^2)$
 $= 0.01689 \text{ kg} \cdot \frac{\text{m}}{\text{s}^2} = 0.01689 \text{ (N)}$ — Ans

3) $h_{x=1\text{m}} = ?$

$Nu_{x=1\text{m}}$ do not forget to use the correct formula for the boundary layer thickness

From eqn (6.26) $\Rightarrow Nu_x = 0.332 Pr^{\frac{1}{3}} Re_x^{\frac{1}{2}}$

So $Re_{x=1\text{m}} = \frac{V_\infty \cdot x}{\nu} = \frac{5 \text{ (m/s)} \cdot 1 \text{ (m)}}{20.92 \times 10^{-6}} = 2.39 \times 10^5$

$\therefore Nu_x = 0.332 (0.7^{\frac{1}{3}}) (2.39 \times 10^5)^{\frac{1}{2}} = 144.1$

$$\text{am } Nu_x = \frac{h_x \cdot x}{k_f}$$

$$\therefore h_x = \frac{Nu_x \cdot k_f}{x} = \frac{144.1 \times (30 \times 10^{-3} \text{ [} \frac{\text{W}}{\text{m} \cdot \text{K}} \text{])}{1 \text{ [m]}}$$

$$\therefore h_{x=1\text{m}} = 4.323 \text{ [} \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \text{]} \quad \underline{\underline{\text{Any}}}$$

$$\textcircled{4} \text{ am } \frac{\delta}{\delta_t} = Pr^{\frac{1}{3}} \quad (\text{ammon (6.27)})$$

$$\delta_t = \frac{5x}{Pr^{\frac{1}{3}} \sqrt{Re_x}}$$

$$\therefore \delta_t = \frac{5 \cdot (1 \text{ [m]})}{(0.7)^{\frac{1}{3}} \cdot \sqrt{2.39 \times 10^5}} = 0.01152 \text{ [m]}$$

$$\textcircled{5} \delta = Pr^{\frac{1}{3}} \cdot \delta_t = (0.7)^{\frac{1}{3}} \cdot 0.01152 \text{ [m]}$$

$$= 0.01023 \text{ [m]}$$

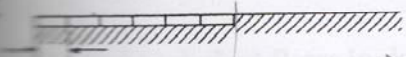
$$\text{ammonoschis (} \delta \text{)} = 0.01023 \text{ m}$$

$$\text{ammonoschis (} \delta_t \text{)} = 0.01152 \text{ m}$$

Example 5.2

Carbon dioxide gas at 400 K moves past a flat plate that consists of 6 strip heaters. The free-stream velocity is 60 m/sec. It is desired to maintain the strip-heater-surface temperature at 600 K. Determine the wattage requirement for each heater to do this. Each heater is 40 mm wide and has an effective heating length (into the page) of 160 mm.

$V_\infty = 60 \text{ m/s}$
 $T_\infty = 400 \text{ K}$
 $T_s = 600 \text{ K}$



$$L = 6 \times (0.04 \text{ m}) = 0.24 \text{ m}$$

$$Re_L = \frac{V_\infty L}{\nu}$$

$$w = 0.16 \text{ m}$$

Temperature of CO_2 $T_{avg} = \frac{400 + 600}{2} = 500 \text{ [K]}$

$$= 500 \text{ [K]}, \nu = 21.8 \times 10^{-6} \left[\frac{m^2}{s} \right], \rho = 1.0594 \left[\frac{kg}{m^3} \right], k_f = 32.5 \times 10^{-3} \left[\frac{W}{m \cdot K} \right]$$

$$Pr = 0.725$$

$$Re_L = \frac{60 \left[\frac{m}{s} \right] \cdot 0.24 \left[m \right]}{21.8 \times 10^{-6} \left[\frac{m^2}{s} \right]} = 6.605 \times 10^5 > Re_{crit}$$

Flow is turbulent

Use the correlation for turbulent flow over a flat plate

$$\bar{Nu}_L = 0.0359 Re_L^{4/5} Pr^{1/3} - 830 Pr^{1/3} \quad ; \quad 0.6 < Pr < 60$$

$$5 \times 10^5 < Re_L < 5 \times 10^7$$

$$0.6 < Pr = 0.725 < 60$$

$$5 \times 10^5 < Re_L = 6.605 \times 10^5 < 5 \times 10^7$$

$$\bar{Nu}_L = 0.0359 (6.605 \times 10^5)^{4/5} (0.725)^{1/3} - 830 (0.725)^{1/3}$$

$$= 1,260$$

$$\bar{h}_L = \frac{\bar{Nu}_L \cdot k_f}{L} = \frac{1,260 \cdot (32.5 \times 10^{-3} \left[\frac{W}{m \cdot K} \right])}{0.24 \left[m \right]} = 170.6 \left[\frac{W}{m^2 \cdot K} \right]$$

$$Q = \bar{h}_L \cdot A \cdot (T_w - T_\infty)$$

$$= 170.6 \left[\frac{W}{m^2 \cdot K} \right] \cdot (0.24 \times 0.16) \left[m^2 \right] \cdot (600 - 400) \left[K \right]$$

$$= 1,310 \text{ [W]} \quad \underline{\underline{Ans}}$$