Chapter 5
Control Chart for Variable

แผนภูมิควบคุมสำหรับตัวแปรชนิดแปรผัน

Control Chart for Variable

- ❑ Introduction
- ❑ Control Chart Techniques
- ❑ State of Control
- ❑ Specifications
- ❑ Process Capability
- ❑ Different Control Charts
- ❑ Other Charts
- ❑ Computer Program

Introduction

Law of Nature: no two natural items in any category are the same

No two objects are ever made exactly alike

- *Variation*
  1. **Within-piece variation ex surface roughness**
  2. **Piece-to-piece variation ex light intensity of four consecutive light bulbs**
  3. **Time-to-time variation**

Introduction

- *Variation*

  is present in every process.
- Chance causes (random causes) are inevitable, difficult to detect or identify.
- Assignable causes are large in magnitude and readily identified.

Variation Source

- Equipment : tool wear, machine vibration, workholding-device positioning, hydraulic and electrical fluctuations.
- Material : tensile strength, ductility, thickness, porosity, moisture content
- Environment : Temperature, light, particle size, pressure, humidity
- Operator : method, physical, emotional

Variation Source

- Equipment
- Material
- Environment
- Operator

} **True variation**

Report variation

⇩

Inspection activity

Variation

❑ ***Chance cause สาเหตุจากธรรมชาติของกระบวนการผลิต***

❑ ***Assignable cause สาเหตุเฉพาะ หรือ สาเหตุพิเศษ – large in magnitude***

Introduction

- When only chance causes are present in a process, the process is considered to be in a state of statistical control. It is stable and predictable.
- When an assignable cause of variation is also present, the process is classified as out of control.

The Control Chart Method

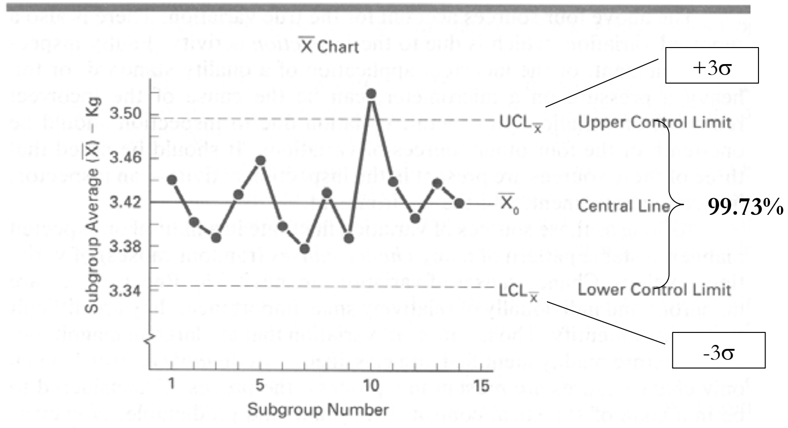


FIGURE 4-1 **Example of a control chart.**

Example

$\overline{X}$ AND R CHART

Work Center Number 365—2
Quality Characteristic Durometer Date 3/6/76

Time	8:30 AM	9:30 AM	10:40 AM	11:50 AM	1:30 PM									
Subgroup	1	2	3	4	5	6	7	8	9	10	11	12	13	14
X_1	55	51	48	45	53									
X_2	52	52	49	43	50									
X_3	51	57	50	45	48									
X_4	53	50	49	43	50									
Sum	211	210	196	176	201									
$\overline{X}$	52.8	52.5	49	44	50.2									
R	4	7	2	2	5									

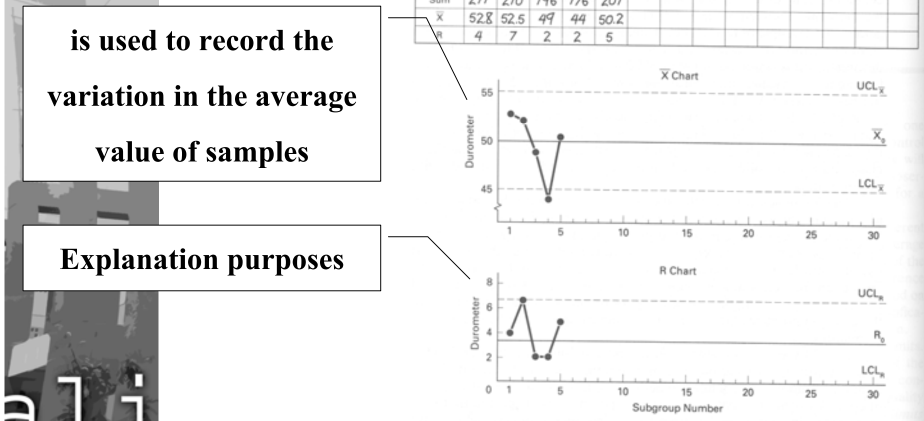


FIGURE 4-2 **Example of a method of reporting inspection results.**

Out of control – subgroup 4

- ❑ **The operator report this fact to supervisor**
- ❑ **Operator and supervisor will then look for an assignable cause**
- ❑ **Take corrective action**

Control chart indicates when and where trouble has occurred.

The Control Chart Method

- Usually an $\overline{X}$ chart for the central tendency and an R chart for the dispersion are used together.
- is a statistical tool that distinguishes between natural and unnatural variation.

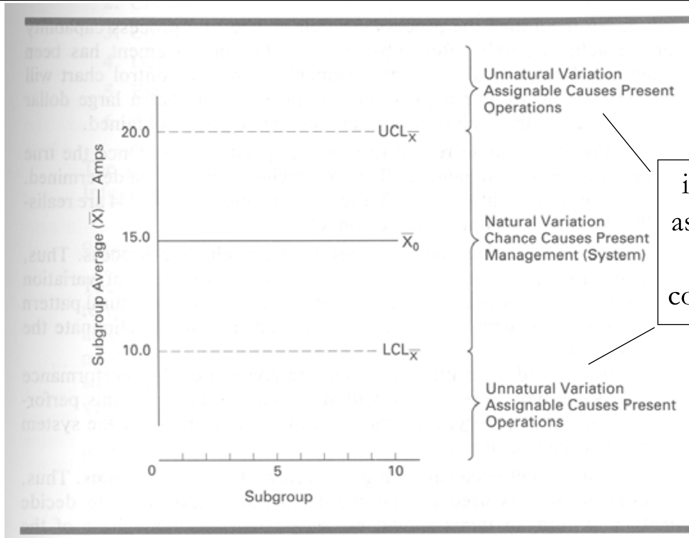


FIGURE 4-3 Natural and unnatural causes of variation.

is the result of assignable cause and require corrective action

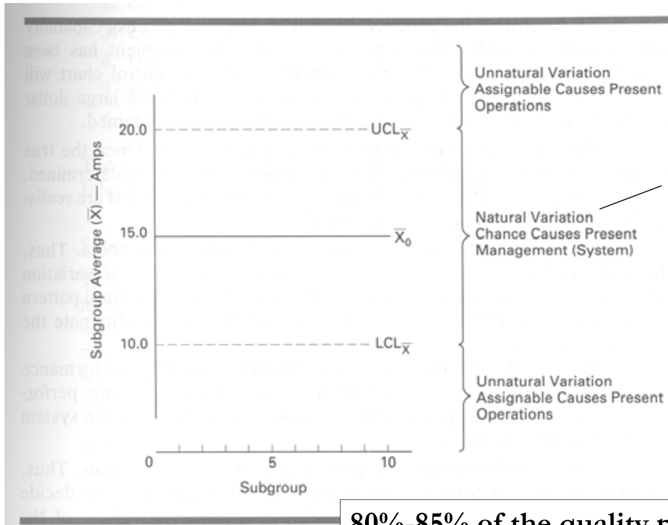


FIGURE 4-3 Natural and unnatural ca

is the result of chance cause. It requires management intervention to achieve quality improvement

80%-85% of the quality problems

are due to management or the system

15%-20% due to operation

Objectives of Variable Control Charts

- For quality improvement
- To determine the true process capability
- For decision in regard to product specifications
- For current decisions in regard to the production process
- For current decisions in regards to recently produced items

Control Chart Techniques

Introduction

The procedure to establish a pair of control charts for the average and the range.

1. **Select the quality characteristic**
2. **Choose the rational subgroup**
3. **Collect the data**
4. **Determine the trial central line and control limits**
5. **Establish the revised central line and control limits**
6. **Achieve the objective**

Select the Quality Characteristic

- must be measurable and can be expressed in numbers.
- can be expressed in term of the seven basic units: length, mass, time, electrical current, temperature, substance, or luminous intensity

  or any derived units, such as power, velocity, force, energy, density, and pressure.

Choose the Rational Subgroup

- The first scheme is to select the subgroup samples from product produced at one instant of time or as close to that instant as possible.
- The second scheme is to select product produced over a period of time so that it is representative of all product.

Instant-time method	Period-of-time method
❖**Have minimum variation within the subgroup and maximum variation among the subgroup**	❖**Have maximum variation within the subgroup and a minimum variation among the subgroup**
❖**One most commonly used since it provides a particular time reference**	❖**Provide better overall results and, therefore, present a more accurate picture of quality**

Guidelines for decision on the size of sample

1. As the subgroup size increases, the control limits become closer to the central value, which makes the control chart more sensitive to small variations in the process average.
2. As the subgroup size increases, the inspection cost per subgroup increase.
3. When destructive testing is used and the item is expensive, a small subgroup size of 2 or 3 is necessary.

Guidelines for decision on the size of sample

4. Because of the ease of computation a sample size of 5 is quite common in industry.
5. From a statistical basis a distribution of subgroup averages, $\bar{X}$'s, are nearly normal for subgroup of 4 of more.
6. When the subgroup size exceed 10, the *s* chart should be used instead of the *R* chart for the control of the dispersion.

***s* is sample standard deviation**

TABLE 4-1 Sample Sizes (From MIL-STD-414/Z1.9, Normal Inspection, Level II).

LOT SIZE	SAMPLE SIZE
91–150	10
151–280	15
281–400	20
401–500	25
501–1,200	35
1,201–3,200	50
3,201–10,000	75
10,001–35,000	100
35,001–150,000	150

produce 4000 pieces/day, 75 sample are suggested.

Subgroup size of 4 - 19 subgroups

Collect the Data

- **May use the type form shown in Figure 4.2. An alternative method is shown in Table 4-2.**

$\bar{X}$ AND R CHART

Work Center Number 365–2
Quality Characteristic Durometer Date 3/6/76

Time	8:30 AM	9:30 AM	10:40 AM	11:50 AM	1:30 PM									
Subgroup	1	2	3	4	5	6	7	8	9	10	11	12	13	14
X_1	55	51	48	45	53									
X_2	52	52	49	43	50									
X_3	51	57	50	45	48									
X_4	53	50	49	43	50									
Sum	211	210	196	176	201									
$\bar{X}$	52.8	52.5	49	44	50.2									
R	4	7	2	2	5									

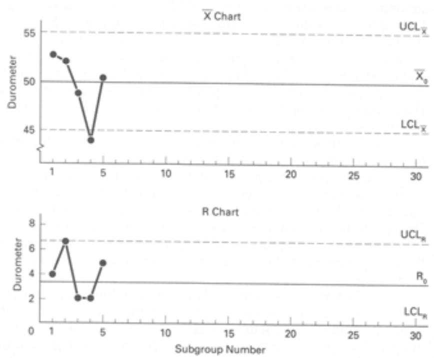


FIGURE 4-2 Example of a method of reporting inspection results.

TABLE 4-2 Data on the Depth of the Keyway (millimeters).[a]

SUBGROUP NUMBER	DATE	TIME	MEASUREMENTS X_1	X_2	X_3	X_4	AVERAGE $\overline{X}$	RANGE R	COMMENT
1	12/23	8:50	35	40	32	37	6.36	0.08	
2		11:30	46	37	36	41	6.40	0.10	
3		1:45	34	40	34	36	6.36	0.06	
4		3:45	69	64	68	59	6.65	0.10	New, temporary
5		4:20	38	34	44	40	6.39	0.10	operator
6	12/27	8:35	42	41	43	34	6.40	0.09	
7		9:00	44	41	41	46	6.43	0.05	
8		9:40	33	41	38	36	6.37	0.08	
9		1:30	48	44	47	45	6.46	0.04	
10		2:50	47	43	36	42	6.42	0.11	
11	12/28	8:30	38	41	39	38	6.39	0.03	
12		1:35	37	37	41	37	6.38	0.04	
13		2:25	40	38	47	35	6.40	0.12	
14		2:35	38	39	45	42	6.41	0.07	
15		3:55	50	42	43	45	6.45	0.08	
16	12/29	8:25	33	35	29	39	6.34	0.10	
17		9:25	41	40	29	34	6.36	0.12	
18		11:00	38	44	28	58	6.42	0.30	Damaged oil line
19		2:35	35	41	37	38	6.38	0.06	
20		3:15	56	55	45	48	6.51	0.11	Bad material
21	12/30	9:35	38	40	45	37	6.40	0.08	
22		10:20	39	42	35	40	6.39	0.07	
23		11:35	42	39	39	36	6.39	0.06	
24		2:00	43	36	35	38	6.38	0.08	
25		4:25	39	38	43	44	6.41	0.06	
Sum							160.25	2.19	

[a] For simplicity in recording, the individual measurements are coded from 6.00 mm.

Because of difficulty in the assembly of a gear hub to a shaft using a key and keyway, the project team recommends using an $\overline{X}$ and R chart. The quality characteristic is the shaft keyway depth of 6.35 mm.

It is necessary to collect a minimum of 25 subgroups of data. A fewer number of subgroups would not provide a sufficient amount of data for the accurate computation of the control limits; and a larger number of subgroups would delay the introduction of the control chart.

Determine the Trial Control Limits

The central lines for the $\overline{X}$ and R charts are obtained using the formulas

$$\overline{\overline{X}} = \frac{\sum_{i=1}^{g} \overline{X}_i}{g} \qquad \overline{R} = \frac{\sum_{i=1}^{g} R_i}{g}$$

Trial control limits for the charts are established at 63 standard deviations from the central value,

$$UCL_{\bar{X}} = \bar{\bar{X}} + 3\sigma_{\bar{X}}$$
$$LCL_{\bar{X}} = \bar{\bar{X}} - 3\sigma_{\bar{X}}$$
$$UCL_R = \bar{R} + 3\sigma_R$$
$$LCL_R = \bar{R} - 3\sigma_R$$

In practice

The calculations are simplified by using factor A_2, D_3, and D_4.

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}$$
$$LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$$
$$UCL_R = D_4\bar{R}$$
$$LCL_R = D_3\bar{R}$$

Table B

TABLE B Factors for Computing Central Lines and 3σ Control Limits for $\bar{X}$, s and R Charts.

OBSERVATIONS IN SAMPLE, n	CHART FOR AVERAGES: FACTORS FOR CONTROL LIMITS			CHART FOR STANDARD DEVIATIONS: FACTOR FOR CENTRAL LINE	CHART FOR STANDARD DEVIATIONS: FACTORS FOR CONTROL LIMITS				CHART FOR RANGES: FACTOR FOR CENTRAL LINE	CHART FOR RANGES: FACTORS FOR CONTROL LIMITS				
	A	A_2	A_3	c_4	B_3	B_4	B_5	B_6	d_2	d_1	D_1	D_2	D_3	D_4
2	2.121	1.880	2.659	0.7979	0	3.267	0	2.606	1.128	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	0	2.568	0	2.276	1.693	0.888	0	4.358	0	2.574
4	1.500	0.729	1.628	0.9213	0	2.266	0	2.088	2.059	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	0	2.089	0	1.964	2.326	0.864	0	4.918	0	2.114
6	1.225	0.483	1.287	0.9515	0.030	1.970	0.029	1.874	2.534	0.848	0	5.078	0	2.004
7	1.134	0.419	1.182	0.9594	0.118	1.882	0.113	1.806	2.704	0.833	0.204	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	0.185	1.815	0.179	1.751	2.847	0.820	0.388	5.306	0.136	1.864
9	1.000	0.337	1.032	0.9693	0.239	1.761	0.232	1.707	2.970	0.808	0.547	5.393	0.184	1.816
10	0.949	0.308	0.975	0.9727	0.284	1.716	0.276	1.669	3.078	0.797	0.687	5.469	0.223	1.777
11	0.905	0.285	0.927	0.9754	0.321	1.679	0.313	1.637	3.173	0.787	0.811	5.535	0.256	1.744
12	0.866	0.266	0.886	0.9776	0.354	1.646	0.346	1.610	3.258	0.778	0.922	5.594	0.283	1.717
13	0.832	0.249	0.850	0.9794	0.382	1.618	0.374	1.585	3.336	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	0.406	1.594	0.399	1.563	3.407	0.763	1.118	5.696	0.328	1.672
15	0.775	0.223	0.789	0.9823	0.428	1.572	0.421	1.544	3.472	0.756	1.203	5.741	0.347	1.653
16	0.750	0.212	0.763	0.9835	0.448	1.552	0.440	1.526	3.532	0.750	1.282	5.782	0.363	1.637
17	0.728	0.203	0.739	0.9845	0.466	1.534	0.458	1.511	3.588	0.744	1.356	5.820	0.378	1.622
18	0.707	0.194	0.718	0.9854	0.482	1.518	0.475	1.496	3.640	0.739	1.424	5.856	0.391	1.608
19	0.688	0.187	0.698	0.9862	0.497	1.503	0.490	1.483	3.689	0.734	1.487	5.891	0.403	1.597
20	0.671	0.180	0.680	0.9869	0.510	1.490	0.504	1.470	3.735	0.729	1.549	5.921	0.415	1.585

Example Problem

To obtain the trial control limits and the central line, the data in Table 4-2 concerning the depth of the shaft keyway will be used. From Table 4-2, the Σx = 160.25, Σ***R*** = 2.19, and ***g*** = 25; thus, the central line are

$$\bar{\bar{X}} = \frac{\sum_{i=1}^{g} \bar{X}_i}{g} \quad \textbf{= 160.25/25 = 6.41 mm}$$

$$\bar{R} = \frac{\sum_{i=1}^{g} R_i}{g} \quad \textbf{= 2.19/25 = 0.0876 mm}$$

Table B

TABLE B Factors for Computing Central Lines and 3σ Control Limits for $\overline{X}$, s and *R* Charts.

OBSERVATIONS IN SAMPLE, *n*	CHART FOR AVERAGES — FACTORS FOR CONTROL LIMITS			CHART FOR STANDARD DEVIATIONS — FACTOR FOR CENTRAL LINE	CHART FOR STANDARD DEVIATIONS — FACTORS FOR CONTROL LIMITS				CHART FOR RANGES — FACTOR FOR CENTRAL LINE	CHART FOR RANGES — FACTORS FOR CONTROL LIMITS				
	A	A_2	A_3	c_4	B_3	B_4	B_5	B_6	d_2	d_3	D_1	D_2	D_3	D_4
2	2.121	1.880	2.659	0.7979	0	3.267	0	2.606	1.128	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	0	2.568	0	2.276	1.693	0.888	0	4.358	0	2.574
4	1.500	0.729	1.628	0.9213	0	2.266	0	2.088	2.059	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	0	2.089	0	1.964	2.326	0.864	0	4.918	0	2.114
6	1.225	0.483	1.287	0.9515	0.030	1.970	0.029	1.874	2.534	0.848	0	5.078	0	2.004
7	1.134	0.419	1.182	0.9594	0.118	1.882	0.113	1.806	2.704	0.833	0.204	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	0.185	1.815	0.179	1.751	2.847	0.820	0.388	5.306	0.136	1.864
9	1.000	0.337	1.032	0.9693	0.239	1.761	0.232	1.707	2.970	0.808	0.547	5.393	0.184	1.816
10	0.949	0.308	0.975	0.9727	0.284	1.716	0.276	1.669	3.078	0.797	0.687	5.469	0.223	1.777
11	0.905	0.285	0.927	0.9754	0.321	1.679	0.313	1.637	3.173	0.787	0.811	5.535	0.256	1.744
12	0.866	0.266	0.886	0.9776	0.354	1.646	0.346	1.610	3.258	0.778	0.922	5.594	0.283	1.717
13	0.832	0.249	0.850	0.9794	0.382	1.618	0.374	1.585	3.336	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	0.406	1.594	0.399	1.563	3.407	0.763	1.118	5.696	0.328	1.672
15	0.775	0.223	0.789	0.9823	0.428	1.572	0.421	1.544	3.472	0.756	1.203	5.741	0.347	1.653
16	0.750	0.212	0.763	0.9835	0.448	1.552	0.440	1.526	3.532	0.750	1.282	5.782	0.363	1.637
17	0.728	0.203	0.739	0.9845	0.466	1.534	0.458	1.511	3.588	0.744	1.356	5.820	0.378	1.622
18	0.707	0.194	0.718	0.9854	0.482	1.518	0.475	1.496	3.640	0.739	1.424	5.856	0.391	1.608
19	0.688	0.187	0.698	0.9862	0.497	1.503	0.490	1.483	3.689	0.734	1.487	5.891	0.403	1.597
20	0.671	0.180	0.680	0.9869	0.510	1.490	0.504	1.470	3.735	0.729	1.549	5.921	0.415	1.585

Trial control limits for the $\overline{X}$ chart are

$$UCL_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R} \quad = 6.41 + (0.729)(0.0876) = 6.47 \text{ mm}$$

$$LCL_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R} \quad = 6.41 - (0.729)(0.0876) = 6.35 \text{ mm}$$

Trial control limits for the ***R*** chart are

$$UCL_R = D_4\overline{R} \quad = (2.282)(0.0876) = 0.20 \text{ mm}$$

$$LCL_R = D_3\overline{R} \quad = (0)(0.0876) = 0 \text{ mm}$$

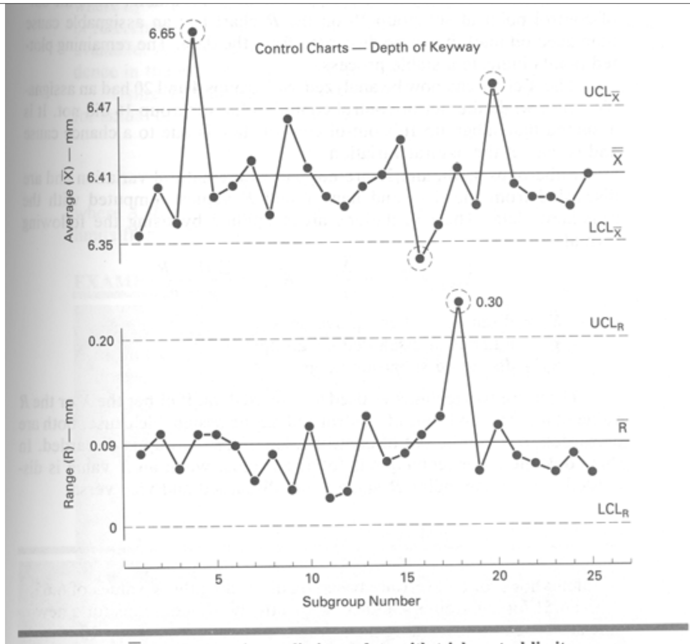


FIGURE 4-4 $\overline{X}$ and *R* chart for preliminary data with trial control limits.

Establish the Revised Control Limits

First step: to post the preliminary data to the chart along with the control limits and central lines.

Next step: to adopt standard values for the central lines, or, more appropriately stated, the best estimate of the standard values with the available data.

- If an analysis of the preliminary data shows good control, then $\bar{\bar{X}}$ and $\bar{R}$ can be considered as representative of the process and these become the standard values, $\bar{X}_0$ and R_0 .
- Good control = no out-of-control points

  = no long runs on either side of the central line

  = no unusual patterns of variation

Most process are not in control when first analyzed.

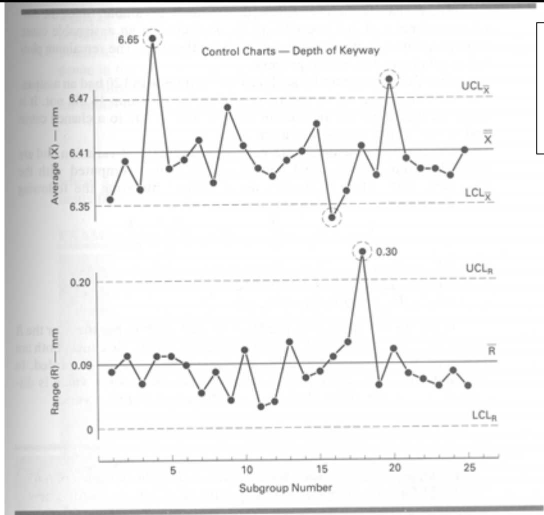


FIGURE 4-4 $\bar{X}$ and R chart for preliminary data with trial control limits.

Out-of-control at subgroups 4, 16, and 20

Out-of-control at subgroup 18

R chart

- Since the out-of-control point at subgroup 18 on the R chart has an assignable cause (damage oil line), it can be discarded from the data.

$\bar{X}$ chart

- Subgroup 4 and 20 had an assignable cause while the out-of-control condition for subgroup 16 did not. It is assumed that subgroup 16's out of control state is due to a chance cause and is part of natural variation.

Subgroups 4, 18, and 20 are not part of the natural variation and are discard from the data and new $\bar{\bar{X}}$ and $\bar{R}$ values computed with the remaining data. The calculations are simplified by using the following formula:

$$\bar{\bar{X}}_{new} = \frac{\sum \bar{X} - \bar{X}_d}{g - g_d} \qquad \bar{R}_{new} = \frac{\sum R - R_d}{g - g_d}$$

Techniques to discard data

- If either the $\bar{X}$ and the R value of a subgroup is out-of-control and has an assignable cause, both are discarded.
- Or only the out-of-control value of a subgroup is discarded.

Example problem (Continued)

Calculations for a new $\bar{X}$ are based on discarding the $\bar{X}$ value of 6.65 and 6.51 for subgroups 4 and 20, respectively. Calculations for a new $\bar{R}$ are based on discarding the ***R*** value of 0.30 for subgroup 18.

$$\bar{\bar{X}}_{new} = \frac{\sum \bar{X} - \bar{X}_d}{g - g_d}$$ **= 160-6.65-6.51/25-2 = 6.40 mm**

$$\bar{R}_{new} = \frac{\sum R - R_d}{g - g_d}$$ **= 2.19-0.30/25-1 = 0.079 mm**

These new values are used to establish the standard values of $\bar{X}_0$, R_0, and σ_0. Thus,

$$\bar{X}_0 = \bar{\bar{X}}_{new}$$

$$R_0 = \bar{R}_{new}$$

$$\sigma_0 = \frac{R_0}{d_2}$$

a factor from Table B

Using the standard values, the central lines and 3σ control limits for actual operations are obtained.

$$UCL_{\bar{X}} = \bar{X}_0 + A\sigma_0$$

$$LCL_{\bar{X}} = \bar{X}_0 - A\sigma_0$$

$$UCL_R = D_2\sigma_0$$

$$LCL_R = D_1\sigma_0$$

Example problem (continued)

From Table B in the Appendix and for a subgroup size of 4, the factors are A = 1.5, d_2 = 2.059, D_1 = 0, and D_2 = 4.698.

$$\bar{X}_0 = \bar{\bar{X}}_{new} = 6.40mm$$

$$R_0 = \bar{R}_{new} = 0.079$$

$$\sigma_0 = \frac{R_0}{d_2} = \frac{0.079}{2.059} = 0.038mm$$

Thus, the control limits are

$$UCL_{\bar{X}} = \bar{X}_0 + A\sigma_0 = 6.40 + (1.5)(0.038) = 6.46mm$$

$$LCL_{\bar{X}} = \bar{X}_0 - A\sigma_0 = 6.40 - (1.5)(0.038) = 6.34mm$$

$$UCL_R = D_2\sigma_0 = (4.698)(0.038) = 0.18mm$$

$$LCL_R = D_1\sigma_0 = (0)(0.038) = 0mm$$

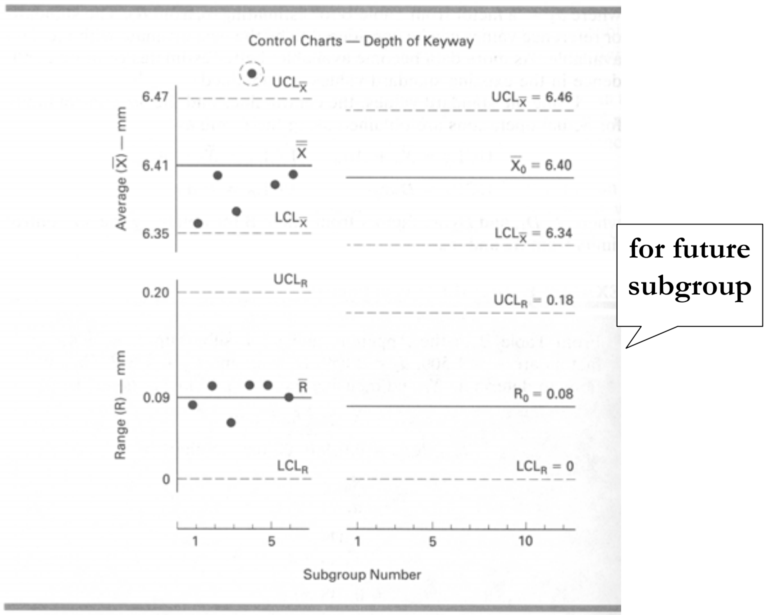


FIGURE 4-5 Trial control limits and revised control limits for $\bar{X}$ and R charts.

Achieving the Objective

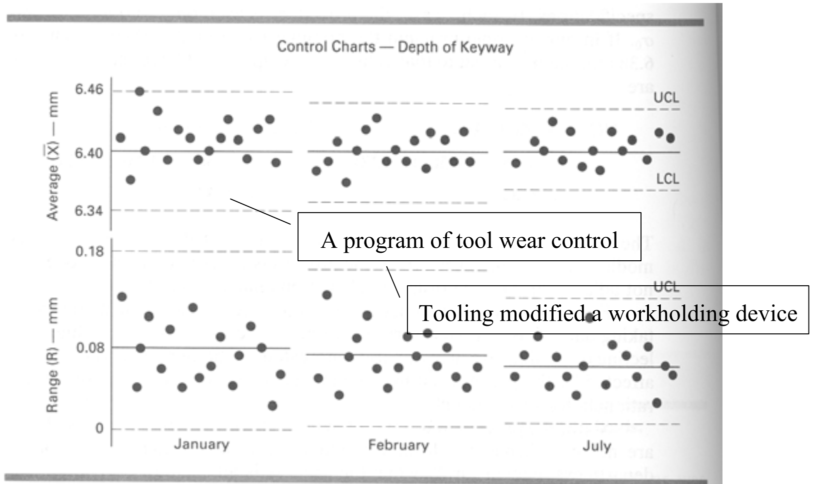


FIGURE 4-6 Continuing use of control charts, showing improved quality.

Question: Establish the $\overline{X}$ and R chart on a certain dimension part (mm). When subgroup size = 6. Determine the trial central line and control limits. Assume assignable cause and revise the central line and limits.

SG	$\overline{X}$	R	SG	$\overline{X}$	R	SG	$\overline{X}$	R
1	20.35	0.34	9	20.48	0.30	17	20.36	0.37
2	20.40	0.36	10	20.42	0.37	18	20.42	0.73
3	20.36	0.32	11	20.39	0.29	19	20.50	0.38
4	20.65	0.36	12	20.38	0.30	20	20.31	0.35
5	20.20	0.36	13	20.40	0.33	21	20.39	0.38
6	20.40	0.35	14	20.41	0.36	22	20.39	0.33
7	20.43	0.31	15	20.45	0.34	23	20.40	0.32
8	20.37	0.34	16	20.34	0.36	24	20.41	0.34
						25	20.40	0.30

The Sample Standard Deviation Control Chart

- An ***R*** chart is easier to compute and easier to explain.
- An ***s*** chart is more accurate than an R chart.
- When subgroup size are less than 10, both charts will graphically portray the same variation.
- As subgroup size increase to 10 or more, extreme values have an undue influence on the ***R*** chart. Therefore, at larger subgroup sizes the ***s*** chart must be used.

R-chart s-chart

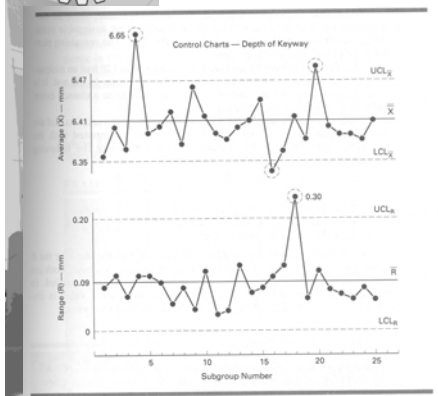


FIGURE 4-4 $\overline{X}$ and R chart for preliminary data with trial control limits.

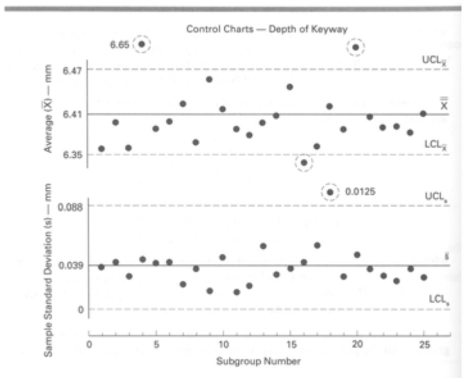


FIGURE 4-7 $\overline{X}$ and s chart for preliminary data with trial control limits.

The step to obtain the $\overline{X}$ and ***s*** chart are the same as for $\overline{X}$ and ***R*** chart except for different formulas.

$$\overline{s} = \frac{\sum_{i=1}^{g} s_i}{g} \qquad UCL_{\overline{X}} = \overline{\overline{X}} + A_3\overline{s}$$

$$LCL_{\overline{X}} = \overline{\overline{X}} - A_3\overline{s}$$

$$\overline{\overline{X}} = \frac{\sum_{i=1}^{g} \overline{X}_i}{g} \qquad UCL_s = B_4\overline{s}$$

$$LCL_s = B_3\overline{s}$$

TABLE 4-3 Data on the Depth of the Keyway (millimeters)[a].

SUBGROUP NUMBER	DATE	TIME	MEASUREMENTS X_1	X_2	X_3	X_4	AVERAGE $\bar{X}$	SAMPLE STANDARD DEVIATION s	COMMENT
1	12/23	8:50	35	40	32	37	6.36	0.034	
2		11:30	46	37	36	41	6.40	0.045	
3		1:45	34	40	34	36	6.36	0.028	
4		3:45	69	64	68	59	6.65	0.045	New, temporary operator
5		4:20	38	34	44	40	6.39	0.042	
6	12/27	8:35	42	41	43	34	6.40	0.041	
7		9:00	44	41	41	46	6.43	0.024	
8		9:40	33	41	38	36	6.37	0.034	
9		1:30	48	44	47	45	6.46	0.018	
10		2:50	47	43	36	42	6.42	0.045	
11	12/28	8:30	38	41	39	38	6.39	0.014	
12		1:35	37	37	41	37	6.38	0.020	
13		2:25	40	38	47	35	6.40	0.051	
14		2:35	38	39	45	42	6.41	0.032	
15		3:55	50	42	43	45	6.45	0.036	
16	12/29	8:25	33	35	29	39	6.34	0.042	
17		9:25	41	40	29	34	6.36	0.067	
18		11:00	38	44	28	58	6.42	0.125	Damaged oil line
19		2:35	35	41	37	38	6.38	0.025	
20		3:15	56	55	45	48	6.51	0.054	Bad material
21	12/30	9:35	38	40	45	37	6.40	0.036	
22		10:20	39	42	35	40	6.39	0.029	
23		11:35	42	39	39	36	6.39	0.024	
24		2:00	43	36	35	38	6.38	0.036	
25		4:25	39	38	43	44	6.41	0.029	
Sum							160.25	0.975	

[a] For simplicity in recording, the individual measurements are coded from 6.00 mm.

Formulas for the computation of the revised control limits using the standard values of $\bar{X}_0$ and σ_0 are

$$\bar{X}_0 = \bar{\bar{X}}_{new} = \frac{\sum \bar{X} - \bar{X}_d}{g - g_d}$$

$$s_0 = \bar{s}_{new} = \frac{\sum s - s_d}{g - g_d}$$

$$\sigma_0 = \frac{s_0}{c_4}$$

$$UCL_{\bar{X}} = \bar{X}_0 + A\sigma_0$$

$$LCL_{\bar{X}} = \bar{X}_0 - A\sigma_0$$

$$UCL_s = B_6\sigma_0$$

$$LCL_s = B_5\sigma_0$$

The first step

To determine the standard deviation for each subgroup from the preliminary data.

For subgroup 1, with value of 6.35, 6.40, 6.32, and 6.37, the standard deviation is

$$s = \sqrt{\frac{n\sum_{i=1}^{n} X_i^{\,2} - \left(\sum_{i=1}^{n} X_i\right)^2}{n(n-1)}} \quad \mathbf{= 0.034\ mm}$$

Example Problem

Using the data of Table 4-3, determine the revised central line and control limits. The first step is to obtain $\bar{X}$ and $\bar{s}$.

$$\bar{s} = \frac{\sum_{i=1}^{g} s}{g} = \frac{0.975}{25} = 0.039mm$$

$$\bar{\bar{X}} = \frac{\sum_{i=1}^{g} \bar{X}_i}{g} = \frac{160.25}{25} = 6.41mm$$

Table B

TABLE B Factors for Computing Central Lines and 3σ Control Limits for $\overline{X}$, s and R Charts.

	CHART FOR AVERAGES			CHART FOR STANDARD DEVIATIONS					CHART FOR RANGES					
OBSERVATIONS IN SAMPLE, n	FACTORS FOR CONTROL LIMITS			FACTOR FOR CENTRAL LINE	FACTORS FOR CONTROL LIMITS				FACTOR FOR CENTRAL LINE	FACTORS FOR CONTROL LIMITS				
	A	A_2	A_3	c_4	B_3	B_4	B_5	B_6	d_2	d_1	D_1	D_2	D_3	D_4
2	2.121	1.880	2.659	0.7979	0	3.267	0	2.606	1.128	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	0	2.568	0	2.276	1.693	0.888	0	4.358	0	2.574
4	1.500	0.729	1.628	0.9213	0	2.266	0	2.088	2.059	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	0	2.089	0	1.964	2.326	0.864	0	4.918	0	2.114
6	1.225	0.483	1.287	0.9515	0.030	1.970	0.029	1.874	2.534	0.848	0	5.078	0	2.004
7	1.134	0.419	1.182	0.9594	0.118	1.882	0.113	1.806	2.704	0.833	0.204	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	0.185	1.815	0.179	1.751	2.847	0.820	0.388	5.306	0.136	1.864
9	1.000	0.337	1.032	0.9693	0.239	1.761	0.232	1.707	2.970	0.808	0.547	5.393	0.184	1.816
10	0.949	0.308	0.975	0.9727	0.284	1.716	0.276	1.669	3.078	0.797	0.687	5.469	0.223	1.777
11	0.905	0.285	0.927	0.9754	0.321	1.679	0.313	1.637	3.173	0.787	0.811	5.535	0.256	1.744
12	0.866	0.266	0.886	0.9776	0.354	1.646	0.346	1.610	3.258	0.778	0.922	5.594	0.283	1.717
13	0.832	0.249	0.850	0.9794	0.382	1.618	0.374	1.585	3.336	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	0.406	1.594	0.399	1.563	3.407	0.763	1.118	5.696	0.328	1.672
15	0.775	0.223	0.789	0.9823	0.428	1.572	0.421	1.544	3.472	0.756	1.203	5.741	0.347	1.653
16	0.750	0.212	0.763	0.9835	0.448	1.552	0.440	1.526	3.532	0.750	1.282	5.782	0.363	1.637
17	0.728	0.203	0.739	0.9845	0.466	1.534	0.458	1.511	3.588	0.744	1.356	5.820	0.378	1.622
18	0.707	0.194	0.718	0.9854	0.482	1.518	0.475	1.496	3.640	0.739	1.424	5.856	0.391	1.608
19	0.688	0.187	0.698	0.9862	0.497	1.503	0.490	1.483	3.689	0.734	1.487	5.891	0.403	1.597
20	0.671	0.180	0.680	0.9869	0.510	1.490	0.504	1.470	3.735	0.729	1.549	5.921	0.415	1.585

trial control limits are

$$UCL_{\overline{X}} = \overline{X}_0 + A_3\sigma_0 = 6.41 + (1.628)(0.039) = 6.47mm$$

$$LCL_{\overline{X}} = \overline{X}_0 - A_3\sigma_0 = 6.41 - (1.628)(0.039) = 6.35mm$$

$$UCL_s = B_4\sigma_0 = (2.266)(0.039) = 0.088mm$$

$$LCL_s = B_3\sigma_0 = (0)(0.039) = 0mm$$

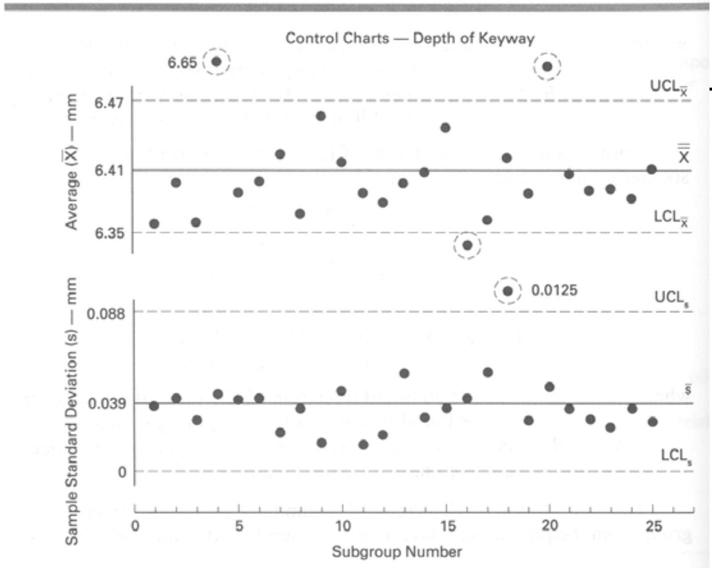


FIGURE 4-7 $\overline{X}$ and s chart for preliminary data with trial control limits.

On the $\overline{X}$ chart, subgroup 4 and 20 have assignable causes, they are discarded.

On the s chart, subgroup 18 is out of control and they have assignable cause, it is discarded.

Computation to obtain the standard values of $\bar{X}_0$, σ_0, and s_0 are as follows:

$$\bar{X}_0 = \bar{\bar{X}}_{new} = \frac{\sum \bar{X} - \bar{X}_d}{g - g_d} = \frac{160.25 - 6.65 - 6.51}{25 - 2} = 6.40mm$$

$$s_0 = \bar{s}_{new} = \frac{\sum s - s_d}{g - g_d} = \frac{0.975 - 0.125}{25 - 1} = 0.0354mm$$

$$\sigma_0 = \frac{s_0}{c_4} = \frac{0.0354}{0.9213} = 0.038mm$$

Compute the revised control limits.

$$UCL_{\bar{X}} = \bar{X}_0 + A\sigma_0 = 6.40 + (1.5)(0.038) = 6.46mm$$

$$LCL_{\bar{X}} = \bar{X}_0 - A\sigma_0 = 6.40 - (1.5)(0.038) = 6.34mm$$

$$UCL_s = B_6\sigma_0 = (2.088)(0.038) = 0.079mm$$

$$LCL_s = B_5\sigma_0 = (0)(0.038) = 0mm$$

Question

Control charts for $\bar{X}$ and ***s*** are maintained on the resistance in ohms of an electrical part. The subgroup size is 6. After 25 subgroups, $\sum \bar{X}$ = 2046.5 and $\sum s$ = 17.4. If the process is in statistical control, what are the control limits and central line.

State of Control

- Process in Control
- Process Out-of-control
- Analysis of Out-of-control Condition

Process in Control

- When the assignable causes have been eliminated from the process to the extent that the point plotted on the control chart remain within the control limits, the process is in a state of control.
- There occurs a natural pattern of variation

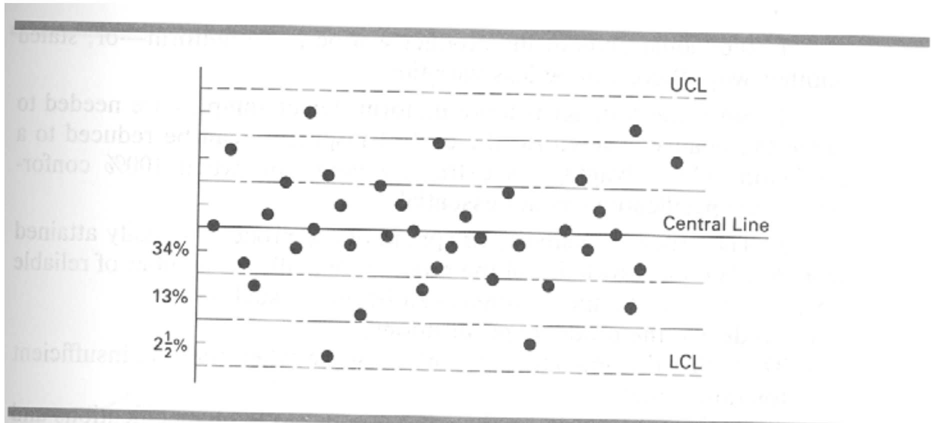
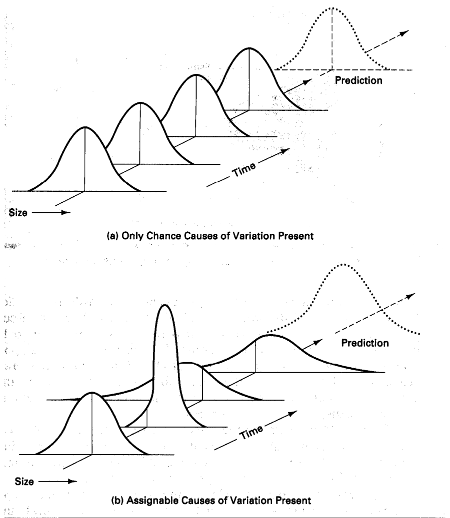


FIGURE 4-8 Natural pattern of variation of a control chart.

Future variation will be the same



RE 4-9 Stable and unstable variation.

Process Out-of-control

- Out-of-control is a change in the process due to an assignable cause.

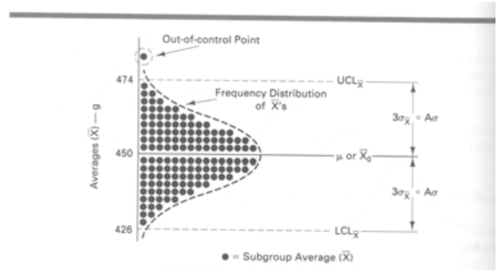
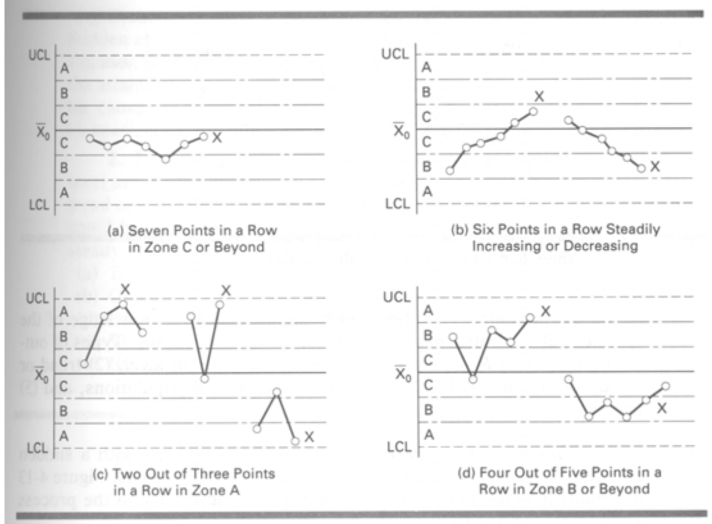


FIGURE 4-10 Frequency distribution of subgroup averages with control limits.



(a) Seven Points in a Row in Zone C or Beyond

(b) Six Points in a Row Steadily Increasing or Decreasing

(c) Two Out of Three Points in a Row in Zone A

(d) Four Out of Five Points in a Row in Zone B or Beyond

FIGURE 4-11 Some unnatural runs—process out-of-control.

Analysis of Out-of-control Condition

- The assignable causes responsible for the condition must be found.
- Type of Out-of-control pattern are
  1. **Change or jump in level**
  2. **Trend or steady change in level**
  3. **Recurring cycles**
  4. **Two population**
  5. **mistake**

1. Change or jump in level

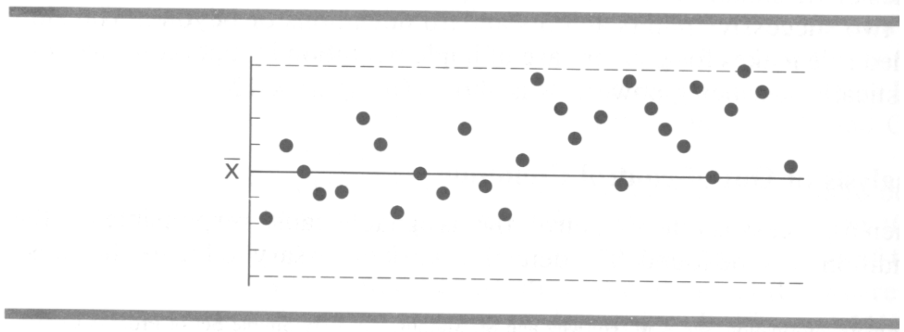


FIGURE 4-13 Out-of-control pattern: change or jump in level.

Change or jump in level

$\overline{X}$ chart, due to

- An intentional or unintentional change in the process setting
- A new or inexperienced operator
- A different raw material
- A minor failure of a machine part

R chart, due to

- Inexperienced operator
- Sudden increase in gear play
- Greater variation in incoming material

Trend or steady change in level

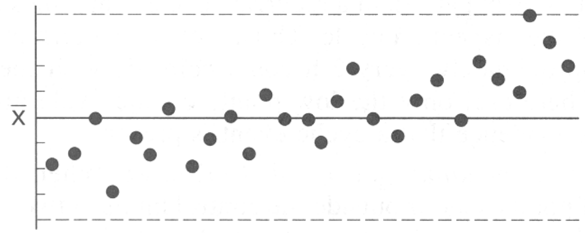


FIGURE 4-14 **Out-of-control pattern: trend or steady change in level.**

Trend or steady change in level

$\bar{X}$ chart, due to

- Tool or die wear
- Gradual deterioration of equipment
- Gradual change in temperature or humidity
- Viscosity breakdown in a chemical process
- Buildup of chips in a work-holding device

R chart, due to

- An improvement in worker skill
- A decrease in worker skill due to fatigue, boredom, inattentions, and so on.
- A gradual improvement in the homogeneity of incoming material

3. Recurring cycles

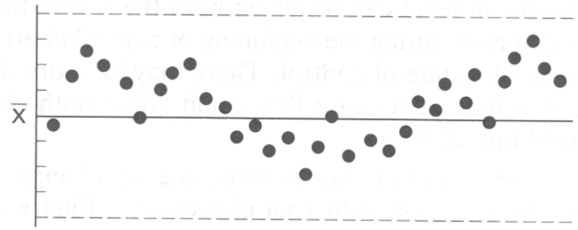


FIGURE 4-15 **Out-of-control pattern: recurring cycles.**

Recurring cycles

$\bar{X}$ chart, due to

- The seasonal effects of incoming material
- The recurring effect of temperature and humidity (cold morning start-up)
- Any daily or weekly chemical, mechanical, or psychological event

R chart, due to

- Operator fatigue and rejuvenation resulting from morning, noon, and afternoon breaks
- Lubrication cycles

4. Two population

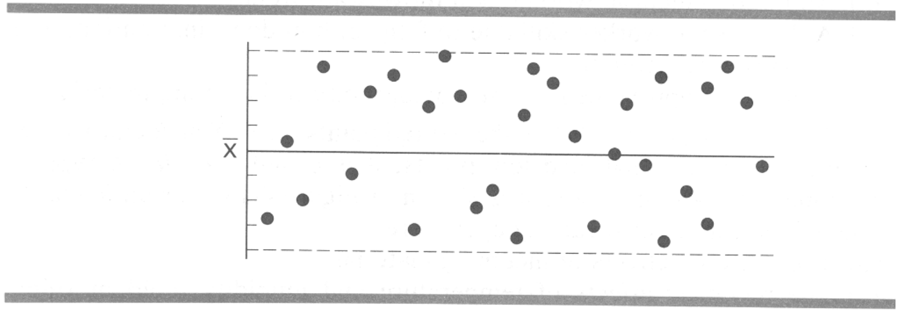


FIGURE 4-16 **Out-of-control pattern: two populations.**

Two population

$\bar{X}$ chart, due to

- ❑ Large differences in material quality
- ❑ two or more machines on the same chart
- ❑ Large differences in test method or equipment

R chart, due to

- ❑ Different workers using the same chart
- ❑ Materials from different suppliers

5. Mistake

Cause are

- ❑ Measuring equipment out of calibration
- ❑ Errors in calculation
- ❑ Errors in using testing equipment
- ❑ Taking samples from different populations

Specification

Individual Values Compared to Averages

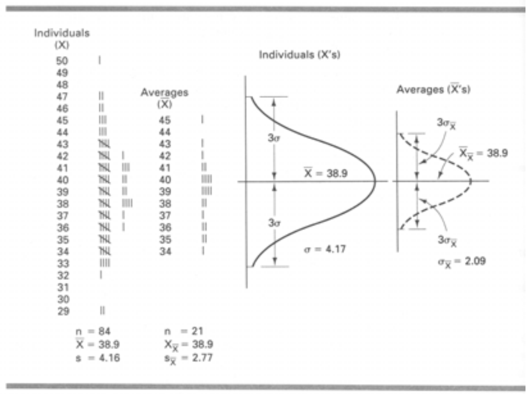


FIGURE 4-17 **Comparison of individual values and averages using the same data.**

Individual Values Compared to Averages

- The averages are grouped much closer to the center than the individual values.
- relationship $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$

Population standard deviation of subgroup average ($\sigma_{\bar{X}}$)

Population standard deviation of individual value (σ)

subgroup size (n)

- The population standard deviation can be estimated from

$$\sigma = \frac{s}{c_4}$$

Table B (c_4)

Central Limit Theorem

If the population from which samples are taken is not normal, the distribution of sample averages will tend toward normality provided that the sample size, n, is at least 4. This tendency get better and better as the sample size gets larger.

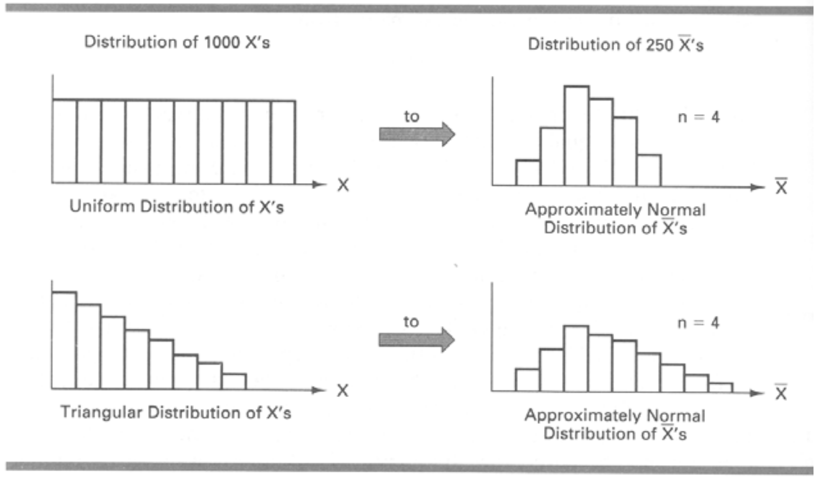


FIGURE 4-18 **Illustration of central limit theorem.**

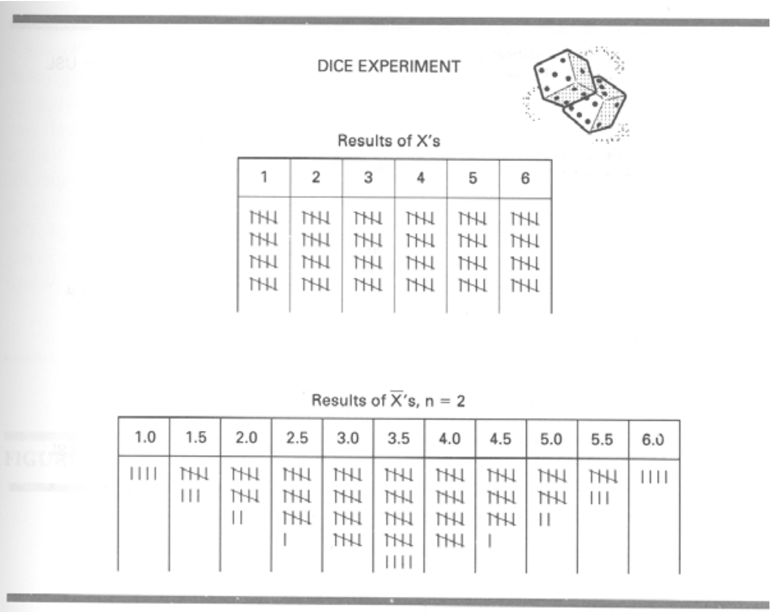


FIGURE 4-19 Dice illustration of central limit theorem.

Control Limits and Specifications

- Control limits are established as a function of the averages; in other words, control limits are for averages.
- Specifications are the permissible variation in the size of the part and are for individual value.
- The specification are established by design engineer to meet a particular function.

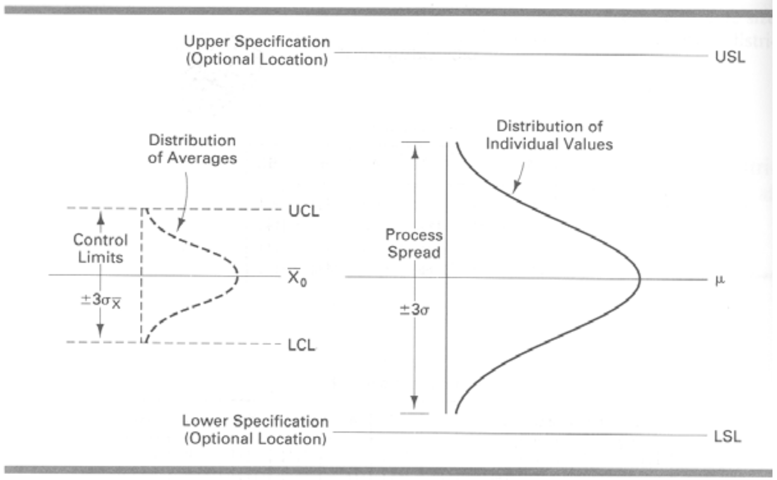
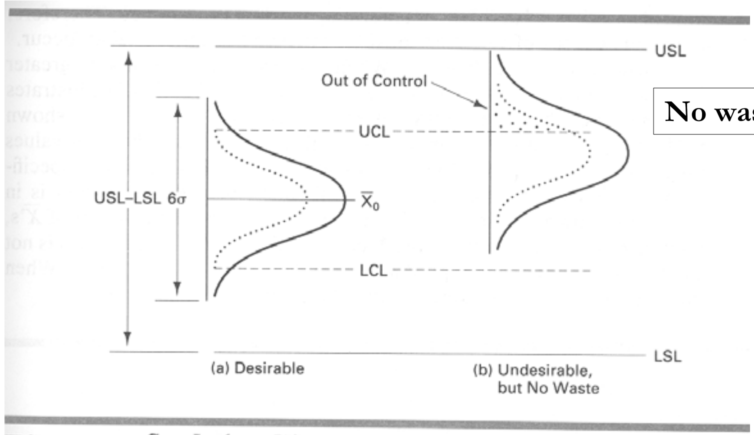


FIGURE 4-20 Relationship of limits, specifications, and distributions.

Process Capability and Tolerance

1. Case I: 6σ < USL-LSL
2. Case II: 6σ = USL-LSL
3. Case III: 6σ > USL-LSL

Case I: 6σ < USL-LSL



No waste

FIGURE 4-21 Case I $6\sigma <$ USL-LSL.

Case II: 6σ = USL-LSL

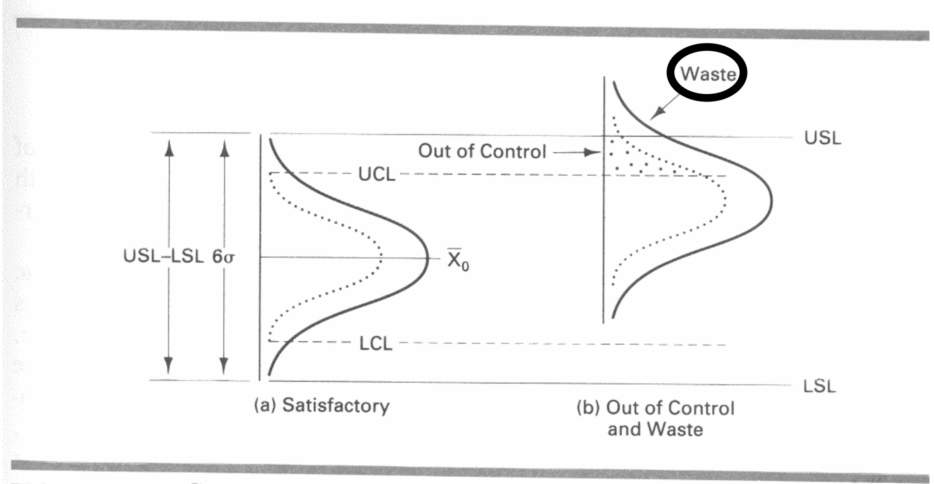


FIGURE 4-22 Case II $6\sigma =$ USL-LSL.

Case III: 6σ > USL-LSL

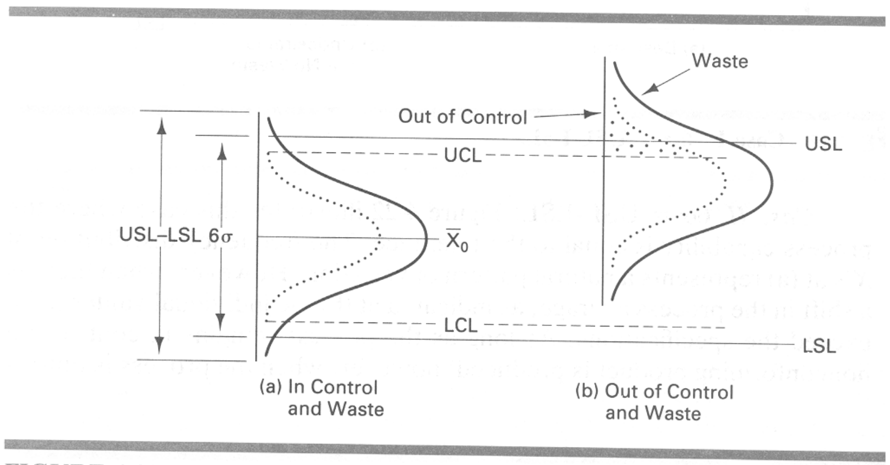


FIGURE 4-23 Case III $6\sigma >$ USL-LSL.

Process Capability

- = 6σ

Does not give the true process capability

- Procedure
  1. **Take 20 subgroups of size 4 for a total of 80 measurements.**
  2. **Calculate the sample standard deviation, *s*, for each subgroup.**
  3. **Calculate the average sample standard deviation,** $\overline{s} = \sum s/g = \sum s/20$
  4. **Calculate the estimate of the population standard deviation.** $\hat{\sigma} = \overline{s}/c_4$
  5. **Process capability will equal** $6\sigma_o$.

Example Problem

- A new process is started and the sum of the sample standard deviations for 20 subgroups of size 4 is 84. Determine the process capability.

Table B

TABLE B Factors for Computing Central Lines and 3σ Control Limits for $\overline{X}$, s and R Charts.

OBSERVATIONS IN SAMPLE, n	CHART FOR AVERAGES: FACTORS FOR CONTROL LIMITS			CHART FOR STANDARD DEVIATIONS: FACTOR FOR CENTRAL LINE	CHART FOR STANDARD DEVIATIONS: FACTORS FOR CONTROL LIMITS				CHART FOR RANGES: FACTOR FOR CENTRAL LINE	CHART FOR RANGES: FACTORS FOR CONTROL LIMITS				
	A	A_2	A_3	c_4	B_3	B_4	B_5	B_6	d_2	d_1	D_1	D_2	D_3	D_4
2	2.121	1.880	2.659	0.7979	0	3.267	0	2.606	1.128	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	0	2.568	0	2.276	1.693	0.888	0	4.358	0	2.574
4	1.500	0.729	1.628	0.9213	0	2.266	0	2.088	2.059	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	0	2.089	0	1.964	2.326	0.864	0	4.918	0	2.114
6	1.225	0.483	1.287	0.9515	0.030	1.970	0.029	1.874	2.534	0.848	0	5.078	0	2.004
7	1.134	0.419	1.182	0.9594	0.118	1.882	0.113	1.806	2.704	0.833	0.204	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	0.185	1.815	0.179	1.751	2.847	0.820	0.388	5.306	0.136	1.864
9	1.000	0.337	1.032	0.9693	0.239	1.761	0.232	1.707	2.970	0.808	0.547	5.393	0.184	1.816
10	0.949	0.308	0.975	0.9727	0.284	1.716	0.276	1.669	3.078	0.797	0.687	5.469	0.223	1.777
11	0.905	0.285	0.927	0.9754	0.321	1.679	0.313	1.637	3.173	0.787	0.811	5.535	0.256	1.744
12	0.866	0.266	0.886	0.9776	0.354	1.646	0.346	1.610	3.258	0.778	0.922	5.594	0.283	1.717
13	0.832	0.249	0.850	0.9794	0.382	1.618	0.374	1.585	3.336	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	0.406	1.594	0.399	1.563	3.407	0.763	1.118	5.696	0.328	1.672
15	0.775	0.223	0.789	0.9823	0.428	1.572	0.421	1.544	3.472	0.756	1.203	5.741	0.347	1.653
16	0.750	0.212	0.763	0.9835	0.448	1.552	0.440	1.526	3.532	0.750	1.282	5.782	0.363	1.637
17	0.728	0.203	0.739	0.9845	0.466	1.534	0.458	1.511	3.588	0.744	1.356	5.820	0.378	1.622
18	0.707	0.194	0.718	0.9854	0.482	1.518	0.475	1.496	3.640	0.739	1.424	5.856	0.391	1.608
19	0.688	0.187	0.698	0.9862	0.497	1.503	0.490	1.483	3.689	0.734	1.487	5.891	0.403	1.597
20	0.671	0.180	0.680	0.9869	0.510	1.490	0.504	1.470	3.735	0.729	1.549	5.921	0.415	1.585

by using the range.

1. Take 20 subgroups of size 4 for a total of 80 measurements.
2. Calculate the range, ***R***, for each subgroup.
3. Calculate the average range, $\bar{R} = \sum R/g = \sum R/20$
4. Calculate the estimate of the population standard deviation. $\hat{\sigma}_0 = \bar{R}/d_2$
5. Process capability will equal $6\sigma_o$.

Example Problem

- An existing process is not meeting the Rockwell-C specifications. Determine the process capability based on the range values for 20 subgroups of size 4. Data are 7, 5, 5, 3, 2, 4, 5, 9, 4, 5, 4, 7, 5, 7, 3, 4, 7, 5, 5, and 7.

Table B

TABLE B Factors for Computing Central Lines and 3σ Control Limits for $\overline{X}$, s and R Charts.

OBSERVATIONS IN SAMPLE, n	CHART FOR AVERAGES: FACTORS FOR CONTROL LIMITS			CHART FOR STANDARD DEVIATIONS: FACTOR FOR CENTRAL LINE	CHART FOR STANDARD DEVIATIONS: FACTORS FOR CONTROL LIMITS				CHART FOR RANGES: FACTOR FOR CENTRAL LINE	CHART FOR RANGES: FACTORS FOR CONTROL LIMITS				
	A	A_2	A_3	c_4	B_3	B_4	B_5	B_6	d_2	d_1	D_1	D_2	D_3	D_4
2	2.121	1.880	2.659	0.7979	0	3.267	0	2.606	1.128	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	0	2.568	0	2.276	1.693	0.888	0	4.358	0	2.574
4	1.500	0.729	1.628	0.9213	0	2.266	0	2.088	2.059	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	0	2.089	0	1.964	2.326	0.864	0	4.918	0	2.114
6	1.225	0.483	1.287	0.9515	0.030	1.970	0.029	1.874	2.534	0.848	0	5.078	0	2.004
7	1.134	0.419	1.182	0.9594	0.118	1.882	0.113	1.806	2.704	0.833	0.204	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	0.185	1.815	0.179	1.751	2.847	0.820	0.388	5.306	0.136	1.864
9	1.000	0.337	1.032	0.9693	0.239	1.761	0.232	1.707	2.970	0.808	0.547	5.393	0.184	1.816
10	0.949	0.308	0.975	0.9727	0.284	1.716	0.276	1.669	3.078	0.797	0.687	5.469	0.223	1.777
11	0.905	0.285	0.927	0.9754	0.321	1.679	0.313	1.637	3.173	0.787	0.811	5.535	0.256	1.744
12	0.866	0.266	0.886	0.9776	0.354	1.646	0.346	1.610	3.258	0.778	0.922	5.594	0.283	1.717
13	0.832	0.249	0.850	0.9794	0.382	1.618	0.374	1.585	3.336	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	0.406	1.594	0.399	1.563	3.407	0.763	1.118	5.696	0.328	1.672
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16	0.750	0.212	0.763	0.9835	0.448	1.552	0.440	1.526	3.532	0.750	1.282	5.782	0.363	1.637
17	0.728	0.203	0.739	0.9845	0.466	1.534	0.458	1.511	3.588	0.744	1.356	5.820	0.378	1.622
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20	0.671	0.180	0.680	0.9869	0.510	1.490	0.504	1.470	3.735	0.729	1.549	5.921	0.415	1.585

Capability index

$$C_p = \frac{USL - LSL}{6\sigma_0}$$

= 1, case II situation.

> 1, case I situation.

< 1, case III situation.

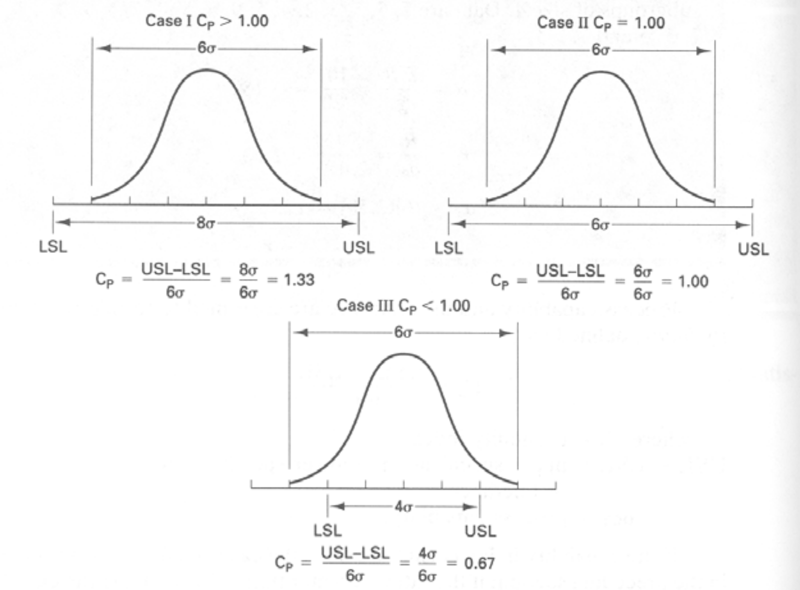

FIGURE 4-24 Capability index and three cases.

Example Problem

- Assume that the specifications are 6.50 and 6.30 in the depth of keyway problem. Determine the capability index before (σ_o= 0.038) and after (σ_o = 0.030) improvement.

Capability ratio

$$C_r = \frac{6\sigma_0}{USL - LSL}$$

Process performance in term of the nominal or target value

$$C_{pk} = \frac{Z(Min)}{3}$$

Where Z(Min) is the smaller of

$$Z(USL) = (USL - \bar{X})/\sigma$$

$$Z(LSL) = (\bar{X} - LSL)/\sigma$$

Example Problem

Determine C_{pk} for the previous example problem (USL=6.50, LSL=6.30, and σ = 0.030) when the average is 6.45.

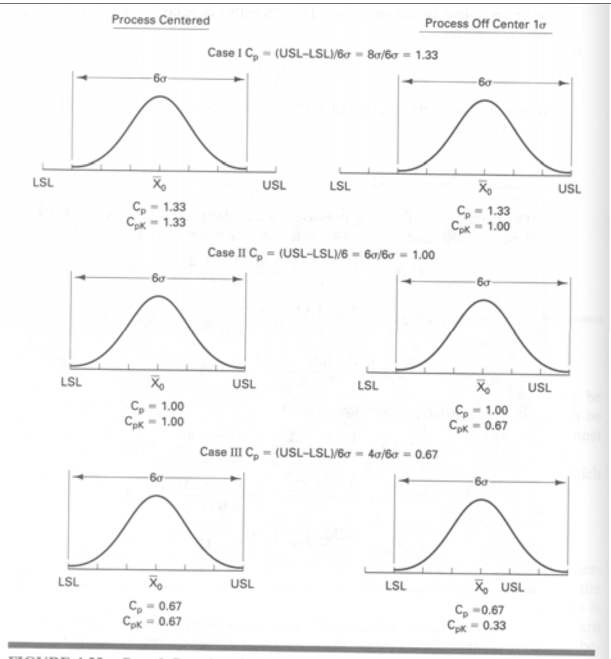

FIGURE 4-25 C_p and C_{pk} values for the three cases.

C_p and C_{pk}

1. The C_p value does not change as the process center changes.
2. $C_p = C_{pk}$ when the process is centered.
3. C_{pk} is always equal to or less then C_p.
4. A C_{pk} value of 1 is a de facto standard. It indicates that the process is producing product that conforms to specification.

5. A C_{pk} value less than 1 is a de facto standard. It indicates that the process is producing product that does not conforms to specification.
6. C_{pk} value less than 1 indicates that the process is not capable.
7. A C_{pk} value of zero indicates the average is equal to one of the specification limits.
8. A negative C_{pk} value indicates that the average is outside the specifications.

Different Control Charts

- Chart for Better Operator Understanding
  1. **Placing individual values on the chart**

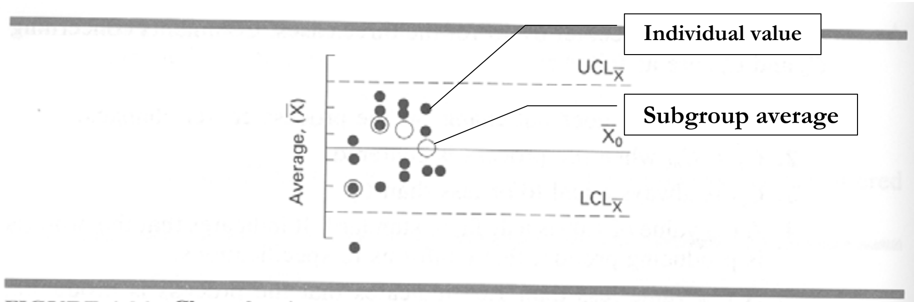

FIGURE 4-26 Chart showing a technique for plotting individual values and subgroup averages.

2. Chart for subgroup sums

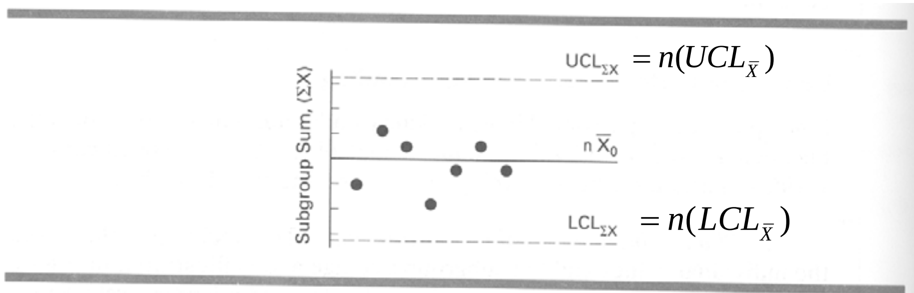

FIGURE 4-27 Subgroup sum chart.

Chart for Variable Subgroup Size

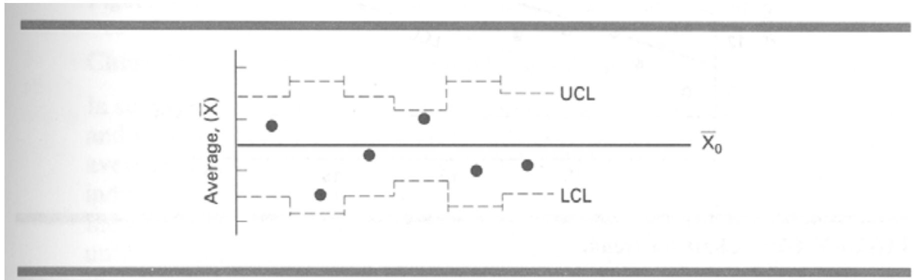

FIGURE 4-28 Chart for variable subgroup size.

Chart for Trends

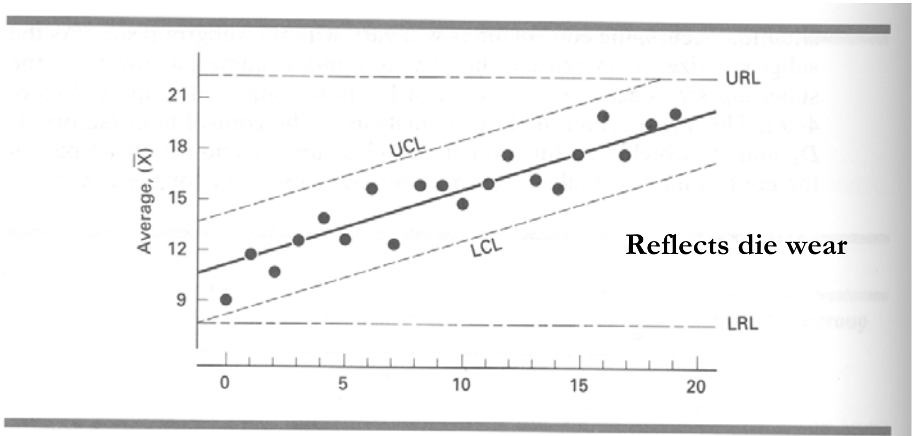

FIGURE 4-29 Chart for trend.

Other Charts

- Charts with Reject Limits

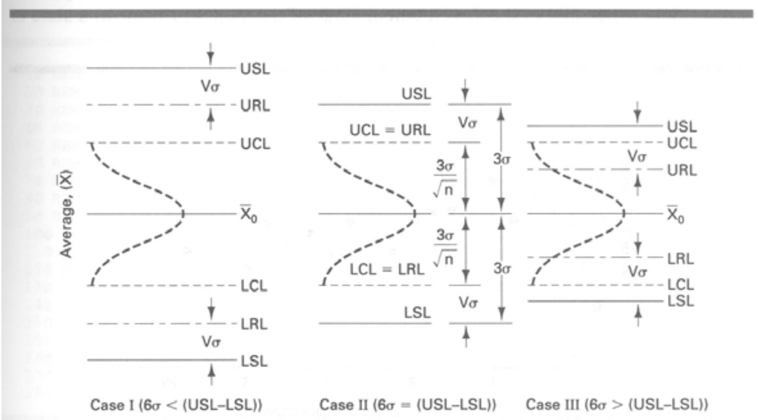

FIGURE 4-32 Relationship of reject limits, control limits, and specifications.

- Run Chart

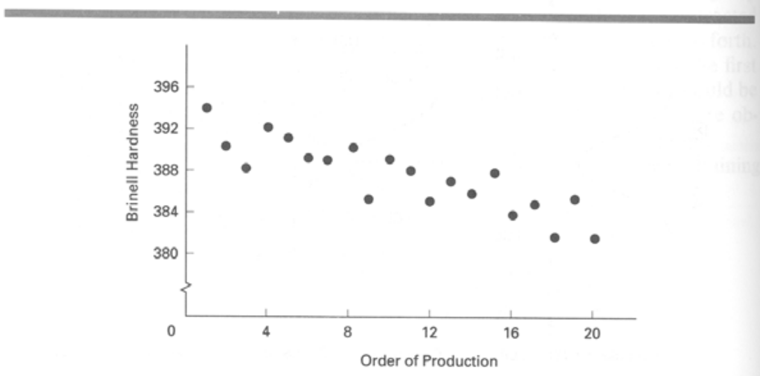

FIGURE 4-33 Run chart for heat treatment operation.

Problem

- A new process is started and the sum of the standard deviations for 20 subgroups of size 4 is 600. If the specifications are 700±80, what is the process capability index? What action would you recommend?

Table B

TABLE B Factors for Computing Central Lines and 3σ Control Limits for $\overline{X}$, s and R Charts.

OBSERVATIONS IN SAMPLE, n	CHART FOR AVERAGES: FACTORS FOR CONTROL LIMITS			CHART FOR STANDARD DEVIATIONS: FACTOR FOR CENTRAL LINE	CHART FOR STANDARD DEVIATIONS: FACTORS FOR CONTROL LIMITS				CHART FOR RANGES: FACTOR FOR CENTRAL LINE	CHART FOR RANGES: FACTORS FOR CONTROL LIMITS				
	A	A_2	A_3	c_4	B_3	B_4	B_5	B_6	d_2	d_1	D_1	D_2	D_3	D_4
2	2.121	1.880	2.659	0.7979	0	3.267	0	2.606	1.128	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	0	2.568	0	2.276	1.693	0.888	0	4.358	0	2.574
4	1.500	0.729	1.628	0.9213	0	2.266	0	2.088	2.059	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	0	2.089	0	1.964	2.326	0.864	0	4.918	0	2.114
6	1.225	0.483	1.287	0.9515	0.030	1.970	0.029	1.874	2.534	0.848	0	5.078	0	2.004
7	1.134	0.419	1.182	0.9594	0.118	1.882	0.113	1.806	2.704	0.833	0.204	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	0.185	1.815	0.179	1.751	2.847	0.820	0.388	5.306	0.136	1.864
9	1.000	0.337	1.032	0.9693	0.239	1.761	0.232	1.707	2.970	0.808	0.547	5.393	0.184	1.816
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13	0.832	0.249	0.850	0.9794	0.382	1.618	0.374	1.585	3.336	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	0.406	1.594	0.399	1.563	3.407	0.763	1.118	5.696	0.328	1.672
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17	0.728	0.203	0.739	0.9845	0.466	1.534	0.458	1.511	3.588	0.744	1.356	5.820	0.378	1.622
18	0.707	0.194	0.718	0.9854	0.482	1.518	0.475	1.496	3.640	0.739	1.424	5.856	0.391	1.608
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20	0.671	0.180	0.680	0.9869	0.510	1.490	0.504	1.470	3.735	0.729	1.549	5.921	0.415	1.585

Problem

- What is the Cpk value when the process average is 700, 740, 780, and 820?