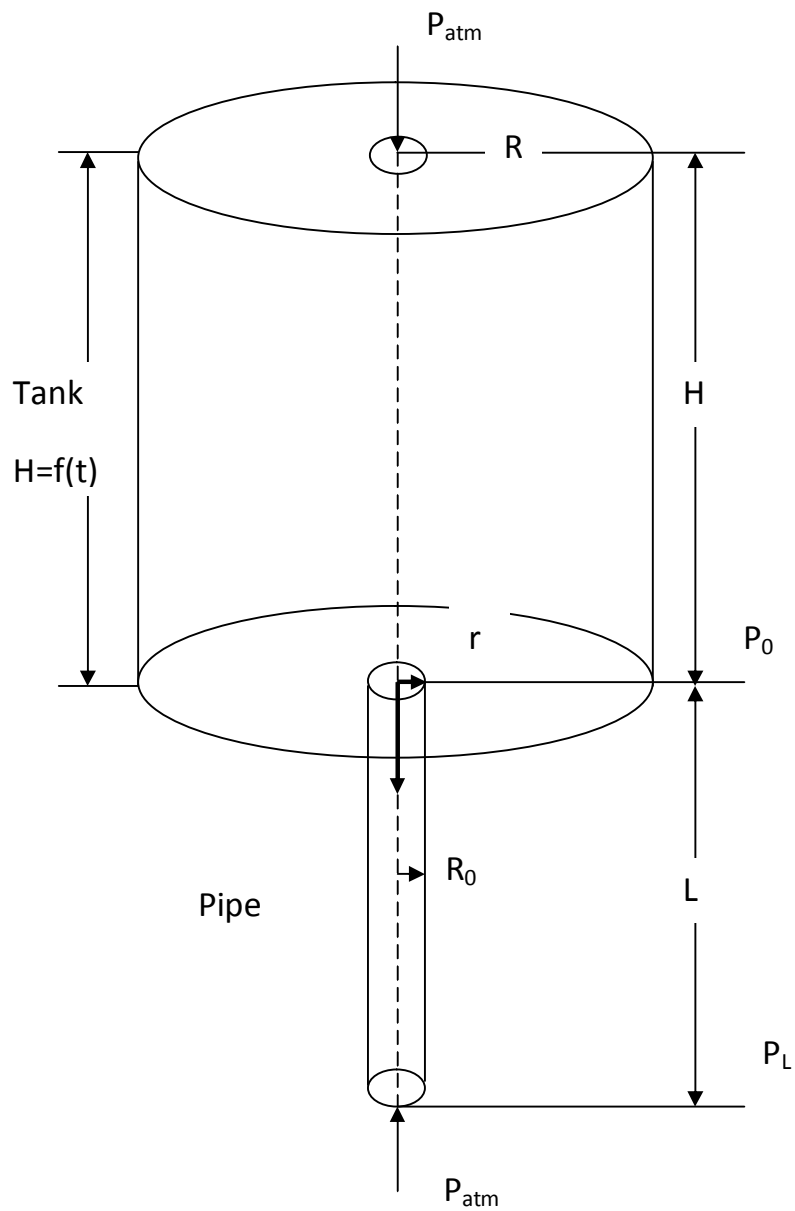


Example 6 Drain water from cylindrical tank



IC: At $t = 0, h = H$ (full tank)

At $t = t_f, h = 0$

Mass balance around tank

(Rate of mass in tank) – (Rate of mass out tank) + (Rate of mass produced by reaction) = (Rate of mass accumulation by time)

$$0 - \rho Q - 0 = \frac{d(\text{mass})}{dt} = \frac{d}{dt}(\rho \cdot \pi R^2 h)$$

Assume incompressible fluid, $\rho = \text{constant}$.

$$Q = -\pi R^2 \frac{dh}{dt} \quad (1)$$

Momentum balance around shell in pipe (from example 3).

$$Q = \frac{\pi R_0^4 (P_0 - P_L + \rho g L)}{8\mu L}$$

$$Q = \frac{\pi R_0^4 (P_{atm} + \rho g h - P_{atm} + \rho g L)}{8\mu L}$$

$$Q = \frac{\pi R_0^4 (\rho g (h + L))}{8\mu L} \quad (2)$$

Set Equation (1) = Equation (2):

$$\frac{\pi R_0^4 (\rho g (h + L))}{8\mu L} = -\pi R^2 \frac{dh}{dt}$$

$$-\int_0^{t_f} \frac{R_0^4 \rho g dt}{8\mu LR^2} = \int_H^0 \frac{dh}{h+L}$$

$$\frac{R_0^4 \rho g t_f}{8\mu LR^2} = -\ln(h+L)|_H^0 = -\ln(L) + \ln(H+L)$$

$$t_f = \left(\frac{8\mu LR^2}{R_0^4 \rho g} \right) \ln\left(1 + \frac{H}{L}\right)$$



Remark: Be careful! H in the problem might not be the full level tank.

Be careful! The tank might contain two immiscible fluids.

