

Wavelet-Based Fractal Analysis of the Epileptic EEG Signal

Suparerk Janjarasjitt* Mary Ann Werz† and Kenneth A. Loparo‡

* Ubon Ratchathani University, Thailand

Case Western Reserve University, USA

E-mail: suparerk.j@ubu.ac.th, suparerk.janjarasjitt@case.edu Tel: +66-4535-3332

† Case Western Reserve University, USA

E-mail: maryann.werz@uhhospitals.org Tel: +1-216-844-3717

‡ Case Western Reserve University, USA

E-mail: kenneth.lopar@case.edu Tel: +1-216-368-4115

Abstract—The wavelet transform is a natural tool for characterizing self-similar signals. In this work, the wavelet-based representation for $1/f$ processes is applied to the intracranial EEG signal of an epilepsy patient to investigate self-similarity characteristics through the spectral exponent γ . The spectral exponent characterizes the distribution of spectral content from low to high frequencies. An increase in the spectral exponent γ leads to sample signals with smoother temporal patterns. Our computational results show that the spectral exponent of the intracranial EEG signal during an epileptic seizure is significantly higher than that associated with other states of the brain, implying that the wavelet-based fractal analysis is potentially a useful computational tool for epileptic seizure detection.

I. INTRODUCTION

Epilepsy is a common brain disorder in which clusters of neurons signal abnormally [1]. More than 50 million individuals worldwide, about 1% of the world's population are affected by epilepsy [2]. In epilepsy, the normal pattern of neuronal activity becomes disturbed, causing strange sensations, emotions, and behavior, or sometimes convulsions, muscle spasms, and loss of consciousness [1]. There are many possible causes of seizures ranging from illness to brain damage to abnormal brain development [1], and epileptic seizures are manifestations of epilepsy [3].

The electroencephalogram (EEG) is a signal that quantifies the electrical activity of the brain, is used to assess and detect brain abnormalities, and is crucial for the diagnosis of epilepsy [1]. Even though the electrical activity of the brain and the EEG patterns during epileptic seizure may differ significantly from the electrical activity during non-seizure period, the detection of epileptic seizures is challenging for a number of reasons.

Several techniques, derived from linear and nonlinear analysis signal analysis, artificial neural networks, etc. have been proposed and evaluated for the detection and classification of epileptic seizures. Recently, concepts and computational tools derived from the contemporary study of complex systems including chaos theory, nonlinear dynamics and fractals have gained increasing interest for applications in biology and medicine because physiological signals and systems can exhibit an extraordinary range of patterns and behaviors [4].

The mathematical concept of a fractal is commonly associated with irregular objects that exhibit a property called self-similarity [5], [4]. Fractal forms are composed of subunits resembling the structure of the macroscopic object [4] which in nature can emerge from statistical scaling behavior in the underlying physical phenomena [6]. The $1/f$ processes are an important class of statistical self-similar random processes [6]. In [7], [8], a wavelet-based representation for $1/f$ processes was developed, and the spectral exponent, γ , that specifies the distribution of power from low to high frequencies can be estimated from the slope of the log-variance of the wavelet coefficients versus the scale.

In this work, the self-similarity characteristics of the intracranial EEG signal of an epilepsy patient is examined using the wavelet-based representation for $1/f$ processes [7], [8]. From the computational results, the spectral exponent γ of the intracranial EEG signal corresponding to various states of the brain, e.g., seizure onset, interictal, pre-ictal, and post-ictal, are distinguishable. Further, the spectral exponent of the EEG signal during an epileptic seizure event is significantly higher than that associated with other states of the brain. Therefore, this suggests that the characteristics of the spectral exponent obtained from wavelet-based fractal analysis may be useful in the detection of epileptic seizure events.

II. BACKGROUND

A. Discrete Wavelet Transform

The discrete wavelet transform (DWT) is a representation of a signal $x(t) \in L_2$ using a countably-infinite set of wavelets that constitute an orthonormal basis [9]. The synthesis and analysis representations of the discrete wavelet transform of the signal $x(t)$ can be expressed as, respectively, [9]

$$x(t) = \sum_m \sum_n d_{m,n} \psi_{m,n}(t) \quad (1)$$

and

$$d_{m,n} = \int_{-\infty}^{\infty} x(t) \psi_{m,n}(t) dt \quad (2)$$

where $\psi(t)$ is a given function, the mother wavelet, and $\{d_{m,n}\}$ are the wavelet coefficients. A family of wavelets

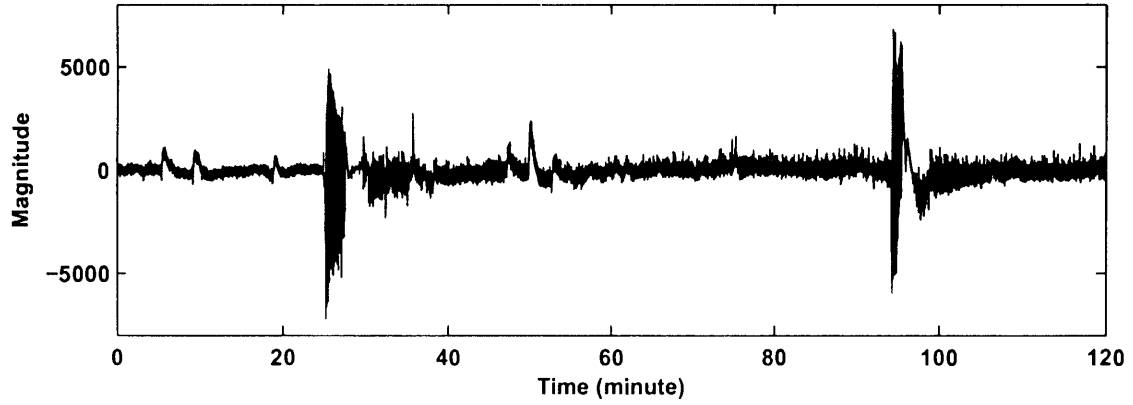


Fig. 1. The intracranial EEG of the epilepsy patient.

$\{\psi_{m,n}(t)\}$ is obtained as normalized dilations and translations of the mother wavelet $\psi(t)$ [10], [11]:

$$\psi_{m,n}(t) = 2^{-m/2} \psi(2^{-m}t - n) \quad (3)$$

where m and n are the dilation and translation indices, respectively. The mother wavelet $\psi(t)$ is localized in both time and frequency [12].

For large scale 2^m , the wavelet $\psi_{m,n}$ is a stretched version of the mother wavelet corresponding to low frequency content, while for small scale 2^m , the wavelet $\psi_{m,n}$ is a contracted version of the mother wavelet corresponding to high frequency content. From a signal processing point of view, the orthonormal wavelet transform can be interpreted as a generalized octave-band filter bank [8], [13] because the mother wavelet $\psi(t)$ is typically an impulse response of a bandpass filter. The orthonormal wavelet transform can also be interpreted in the context of multiresolution analysis (MRA) [11].

B. $1/f$ Processes

In general, models of $1/f$ processes are represented using a frequency domain characterization. The dynamics of $1/f$ processes exhibit power-law behavior [14] and can be characterized in the form of [8]

$$S(\omega) \sim \frac{\sigma_x^2}{|\omega|^\gamma} \quad (4)$$

over several decades of frequency ω , where $S(\omega)$ is the Fourier transform of the signal $x(t)$ and γ is a spectral exponent. An increase in the spectral exponent γ specifying a distribution of spectral content from low to high frequencies leads to sample functions with smoother temporal patterns [8], [6].

C. Wavelet-Based Representation for $1/f$ Processes

The wavelet-based representation for $1/f$ processes developed in [7] is presented in the following theorem.

Theorem 1: [8] Consider any orthonormal wavelet basis with R th-order regularity for some $R \geq 1$. Then the random

process constructed via the expansion

$$x(t) = \sum_m \sum_n d_{m,n} \psi_{m,n}(t) \quad (5)$$

where the $d_{m,n}$ are a collection of mutually uncorrelated, zero-mean random variables with variances

$$\text{var}(d_{m,n}) = \sigma^2 2^{\gamma m} \quad (6)$$

for some parameter $0 < \gamma < 2R$, has a time-averaged spectrum

$$S_x(\omega) = \sigma^2 \sum_m 2^{\gamma m} |\Psi(2^m \omega)|^2 \quad (7)$$

that is nearly $1/f$, i.e.,

$$\frac{\sigma_L^2}{|\omega|^\gamma} \leq S_x(\omega) \leq \frac{\sigma_U^2}{|\omega|^\gamma} \quad (8)$$

for some $0 < \sigma_L^2 \leq \sigma_U^2 < \infty$, and has octave-spaced ripple, i.e., for any integer k

$$|\omega|^\gamma S_x(\omega) = |2^k \omega|^\gamma S_x(2^k \omega). \quad (9)$$

Here, $\Psi(\omega)$ denotes the Fourier transform of the mother wavelet $\psi(t)$.

Accordingly, from Theorem 1, the spectral exponent γ of a $1/f$ process can be determined from the linear relationship between $\log_2 \text{var}(d_{m,n})$ and the level m . The spectral exponent can then be given by

$$\gamma = \frac{\Delta \log_2 \text{var}(d_{m,n})}{\Delta m}. \quad (10)$$

III. ANALYTIC FRAMEWORK

A. Data and Subject

We analyze data from long-term intracranial EEG recordings of an epilepsy patient at University Hospitals of Cleveland, Case Medical Center in Cleveland, Ohio, USA before surgery. With the consent of the patient, intracranial EEG data were recorded for few days using a Nihon-Kohden EEG system with a sampling rate of 1000 Hz and band-pass filter with a passband of 0.10-300 Hz.

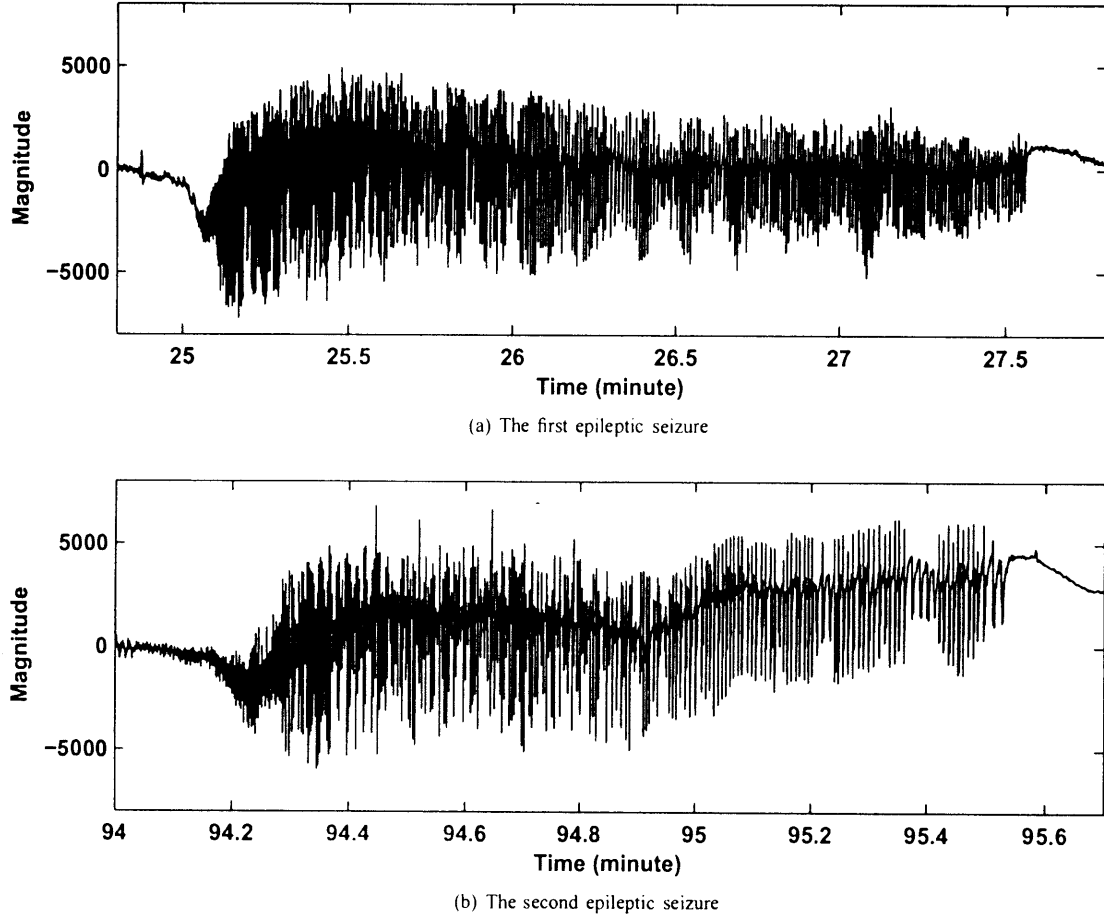


Fig. 2. The segments of the intracranial EEG corresponding to the epileptic seizures.

In this work, only a single channel of a 2-hour record of the intracranial EEG data acquired from within the focal region of the seizures is examined. The intracranial EEG signal is illustrated in Fig. 1. There are 2 seizure events occurring at between 24' 47" and 27' 36", and between 93' 57" and 95' 45", respectively. Note that the first seizure is several hours after the preceding seizure event. In addition, the segments of the intracranial EEG signal corresponding to the first and second epileptic seizures are illustrated in Fig. 2(a) and Fig. 2(b), respectively.

B. The Wavelet-Based Fractal Analysis

In the computational experiment, the intracranial EEG signal is partitioned into epochs of 8000 samples (8 seconds). In the wavelet decomposition, the 5th order Coiflet orthonormal wavelet bases are used. The epochs of the intracranial EEG signal are decomposed into 6 levels. Only the wavelet coefficients of levels $m = 1, 2, 3, 4, 5$, i.e., $\{x_n^1, x_n^2, x_n^3, x_n^4, x_n^5\}$ are used in the estimation of the spectral exponent γ . The spectral exponent, i.e., the linear relationship between $\log_2 \text{var}(d_{m,n})$ and the level m , is estimated using linear least-squares regression estimation.

IV. RESULTS

The spectral exponent γ of the intracranial EEG signal of the epilepsy patient is illustrated in Fig. 3. The spectral exponent γ of the intracranial EEG signal corresponding to various states of the brain has distinguishable characteristics. In particular, we observe that the spectral exponent γ of the intracranial EEG signal during an epileptic seizure is significantly higher than that associated with other states of the brain. Further, the spectral exponent γ of the intracranial EEG signal dramatically increases during seizure onset, suddenly decreases right after the epileptic seizure, and then gradually increases returning to baseline as the influence of the epileptic seizure becomes less intense.

From the computational result shown in Fig. 3, it is obviously that both Using the proposed technique, epileptic seizures can be detected using a simple method such as thresholding. Although the threshold value is likely patient specific and would need to be adjusted based on recorded data. For the current patient, if the spectral exponent $\gamma = 3$ is set as the threshold, the first and the second epileptic seizures are then detected between 25' 9" and 27' 38", and between

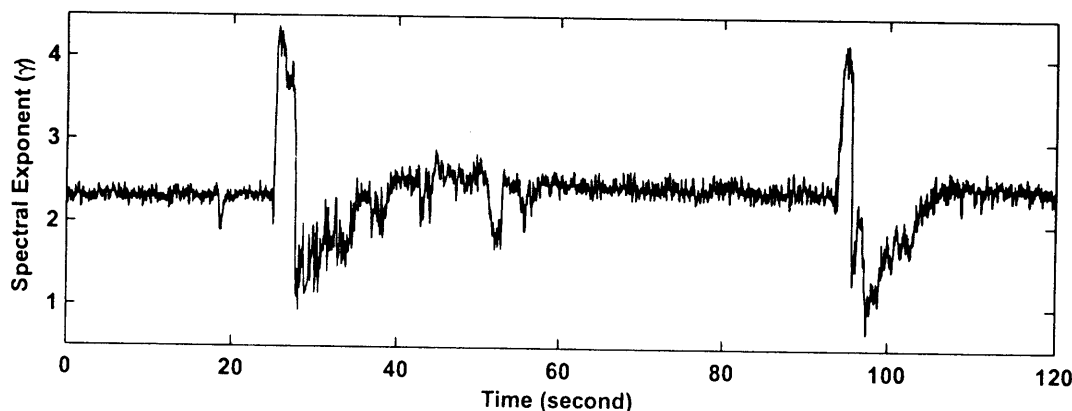


Fig. 3. The spectral exponent γ of the intracranial EEG of the epilepsy patient.

94' 1" and 95' 37", respectively.

V. CONCLUSIONS

Self-similarity characteristics of the intracranial EEG signal of an epilepsy patient are examined using the spectral exponent γ obtained from wavelet-based fractal analysis. From the computational results, it is observed that the intracranial EEG signal during an epileptic seizure exhibits a significantly higher spectral exponent γ than that associated with other states of the brain. Although the results are preliminary and conclusions are limited because only data from a single patient recording has been analyzed, the results do suggest that intracranial EEG signals during epileptic seizure have smoother temporal patterns and less complex temporal characteristics. This is consistent with other findings that suggest that the intracranial EEG is less complex during seizure events.

We hypothesize that the characteristics of the spectral exponent γ of the intracranial EEG signal corresponding to various states of the brain can be used to identify and classify various states of the brain. Future work will further investigate the application of wavelet-based fractal analysis as a computational tool for epileptic seizure detection.

ACKNOWLEDGMENT

Dr. Janjarasjitt is supported by a TRF-CHE Research Grant for New Scholar, jointly funded by the Thailand Research Fund (TRF) and the Commission on Higher Education (CHE), the Ministry of Education, Thailand, under Contract No. MRG5280189.

REFERENCES

- [1] Seizure and Epilepsy: Hope through Research National Institute of Neurological Disorders and Stroke (NINDS), Bethesda, MD, 2004 [Online]. Available: http://www.ninds.nih.gov/disorders/epilepsy/detail_epilepsy.htm
- [2] B. Litt and J. Echauz, "Prediction of epileptic seizures," *Lancet Neurology*, vol. 1, pp. 22-30, 2002.
- [3] A. Subasi, "Epileptic seizure detection using dynamic wavelet network," *Expert Systems with Applications*, vol. 29, pp. 343-355, 2005.
- [4] A. L. Goldberger, "Complex systems," *Proc. Am. Thorac. Soc.*, vol. 3, pp. 467-472, 2006.
- [5] B. B. Mandelbrot, *The Fractal Geometry of Nature*. WH Freeman: San Francisco, 1982.
- [6] G. W. Wornell, *Signal Processing with Fractals: A Wavelet-Based Approach*. Prentice Hall: New Jersey, 1995.
- [7] G. W. Wornell, "A Karhunen-Loève-like expansion for $1/f$ processes via wavelets," *IEEE Trans. Inform. Theory*, vol. 36, pp. 859-861, 1990.
- [8] G. W. Wornell, "Wavelet-based representations for the $1/f$ family of fractal processes," *Proceedings of the IEEE*, vol. 81, pp. 1428-1450, 1993.
- [9] S. Mallat, *A Wavelet Tour of Signal Processing*. Academic Press: San Diego, 1998.
- [10] I. Daubechies, "Orthonormal bases of compactly supported wavelets," *Commun. Pure Appl. Math.*, vol. XLI, pp. 909-996, 1988.
- [11] S. G. Mallat, "A theory for multiresolution signal decomposition: the wavelet representation," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 11, pp. 674-693, 1989.
- [12] A. Cohen and J. Kovacevic, "Wavelets: the mathematical background," *Proceedings of the IEEE*, vol. 84, pp. 514-522, 1996.
- [13] G. W. Wornell and A. V. Oppenheim, "Estimation of fractal signals from noisy measurements using wavelets," *IEEE Trans. Signal Processing*, vol. 40, pp. 611-623, 1992.
- [14] P. A. Watters, "Fractal structure in the electroencephalogram," *Complexity International*, vol. 5, 1998. Available: <http://www.complexity.org.au/ci/vol05/watters/watters.html>

ISPACS 2009

**2009 International Symposium on
Intelligent Signal Processing and
Communication Systems
(ISPACS 2009)**



**December 7-9, 2009
Kanazawa Excel Hotel
Tokyu
Kanazawa Japan**

What's New**Call for Papers****Important Due Dates****Paper Submission****Download****User Site****Registration****Committees****Venue****Kanazawa City****Accommodation**

**2009 International Symposium on
Intelligent Signal Processing and
Communication Systems (ISPACS 2009)
December 7-9, 2009
Kanazawa Excel Hotel Tokyu, Kanazawa Japan**

What's New

September 15, 2009

Review results have been sent to all authors by email. If you had submitted paper and haven't received any notification yet, please, don't hesitate to contact to ispacs2009-secretariat@splab.cs.kitami-it.ac.jp.

August 19, 2009

IEEE PDF eXpress site is now available for ISPACS 2009.

August 18, 2009

Review results will be notified on **September 15, 2009**.

July 14, 2009

IEEE Catalog Number for the Proceedings of ISPACS 2009 is available.

IEEE PDF eXpress site is available for ISPACS 2009 from Sept. 1st, 2009.

July 2, 2009

PROCEEDINGS OF ISPACS 2009 Will Be Included in IEEE Xplore and Indexed by EI.

Please Verify The IEEE Xplore Compatibility of Your Camera Ready Paper

June 30, 2009

Deadline of the paper submission is extended to July 15, 2009.

June 12, 2009

Deadline of the paper submission is extended to June 30, 2009.

February 26, 2009

User Site for paper submission opens.

Author's Kit is available at download page.

Webpage for Paper Submission opens.

Home page of ISPACS 2009 opens.

Sponsors

Notification

Subject: [ISPACS 2009] notification of review result [0029].
From: ispacs2009-secretariat@splab.cs.kitami-it.ac.jp
To: ensupajt@ubu.ac.th, maryann.werz@uhhospitals.org, kenneth.loparo@case.edu
Cc: ispacs2009-secretariat@splab.cs.kitami-it.ac.jp
Reply-To: ispacs2009-secretariat@splab.cs.kitami-it.ac.jp
Return-Path: ispacs2009-secretariat@splab.cs.kitami-it.ac.jp
X-Mailer: PHP 5

Paper ID : 0029
Title : Wavelet-Based Fractal Analysis of the Epileptic EEG Signal
Author(s) : Suparerk Janjarasjitt, Mary Ann Werz, Kenneth A Loparo

Dear Dr. Suparerk Janjarasjitt, Prof. Mary Ann Werz, Prof. Kenneth A Loparo,

Thank you for submitting the above listed manuscript for consideration in the ISPACS 2009 program. We are pleased to inform that your above-referenced manuscript has been accepted for presentation at ISPACS 2009. CONGRATULATIONS! We look forward to your participation and presentation at ISPACS 2009.

For preparing your final camera-ready manuscript, please revise your paper carefully according to the review comments attached below. If there are no comments, your paper is accepted as it is.

Also please read "Guidelines for Preparing Manuscripts" again carefully to fit your manuscript for our publishing style. (cf. download page of ISPACS 2009 website, <http://splab.cs.kitami-it.ac.jp/ispacs2009/?Download>)

Moreover, please verify the IEEE Xplore compatibility of your camera ready paper. If your file does not meet Xplorer compliance, it will NOT be published and will be removed from ISPACS 2009 Proceedings and the IEEE Xplorer system. (cf. submission page of ISPACS 2009 website, <http://splab.cs.kitami-it.ac.jp/ispacs2009/?Submission#IEEE-Compatibility>)

For the sake of publishing your paper in the Proceedings of ISPACS 2009, you are requested to do followings.

- (1) Update the paper information, such as paper title, authors and abstract, at the submission page of the user site

<https://splab.cs.kitami-it.ac.jp/ispacs2009/user>

no later than 15 October, 2009.

- (2) Upload the final PDF file of the manuscript at the submission page of the user site

<https://splab.cs.kitami-it.ac.jp/ispacs2009/user>

no later than 15 October, 2009.

- (3) Send your signed IEEE Copyright Form to ISPACS 2009 Secretariat via Fax

Fax: +81-76-234-4900

or its scanned PDF or image file to

E-mail: ispacs2009-copyright@leo.kanazawa-u.ac.jp

no later than 15 October, 2009. The IEEE Copyright Form can be downloaded via the webpage

<http://splab.cs.kitami-it.ac.jp/ispacs2009/?Download>

- (4) For each paper to be included in the symposium proceedings, AT LEAST ONE AUTHOR PER PAPER MUST REGISTER AT A NON-STUDENT RATE BY 15 October, 2009.

See <http://splab.cs.kitami-it.ac.jp/ispacs2009/?Registration>

Again Please accept our thanks for submitting your paper to the Symposium. We look forward to seeing you in Kanazawa, for an interesting and exciting Symposium.

Sincerely yours,

Hitoshi Kiya, Professor
Chair of ISPACS 2009 Technical Program Committee
ispacs2009-committee@splab.cs.kitami-it.ac.jp

Review Comments:

--- Comments from Reviewer #1 ---

I found several references related to a fractal analysis of EEG.

Xiaoli Li et al, "Fractal spectral analysis of pre-epileptic seizures in terms of criticality", 2005 J. Neural Eng. 2 11-16

E. Pereda et al, "Non-linear behaviour of human EEG: fractal exponent versus correlation dimension in awake and sleep stages"

You need to clarify an originality of your study. I expect further experiments and development of a detection algorithm.

--- Comments from Reviewer #2 ---

1) You should show why the 5th order Coiflet orthonormal wavelet was adopted.

2) You should show the calculational result with a lot more epileptic EEG signal.



บันทึกข้อความ

ส่วนราชการ หน่วยสนับสนุนการวิจัยและบริการ คณะวิศวกรรมศาสตร์ โทร.3319

ที่ ศร 0529.8.1.3/1632

วันที่ 28 ตุลาคม 2552

เรื่อง ขออนุมัติงบประมาณสนับสนุนการนำเสนอบทความวิชาการในการประชุมวิชาการระดับนานาชาติ

เรียน คณบดี

สืบเนื่อง จากมติเวียนคณะกรรมการบริหารงานวิจัย (ERB) คณะวิศวกรรมศาสตร์ โดยได้พิจารณาการนำเสนอผลงานทางวิชาการระดับนานาชาติ ณ ประเทศ ญี่ปุ่น ของ ดร.ศุภฤกษ์ จันทร์จรัสจิตต์ ตามความทราบแล้วนั้น

มติเวียน ของคณะกรรมการบริหารงานวิจัย จำนวน 6 ท่านจากทั้งหมด 9 ท่าน ซึ่งเกินกึ่งหนึ่งขององค์ประชุม ได้เห็นชอบในการสนับสนุนงบประมาณการนำเสนอบทความทางวิชาการระดับนานาชาติ ณ ประเทศญี่ปุ่น ระหว่างวันที่ 7-9 ธันวาคม 2552 ของ ดร.ศุภฤกษ์ จันทร์จรัสจิตต์ อาจารย์สังกัดภาควิชาวิศวกรรมไฟฟ้าและอิเล็กทรอนิกส์ โดยให้การสนับสนุนงบประมาณตามค่าใช้จ่ายจริงไม่เกิน 40,000 บาท (สี่หมื่นบาทถ้วน) ทั้งนี้เป็นไปตามประกาศ คณะวิศวกรรมศาสตร์ ฉบับที่ 38/2549 เรื่อง หลักเกณฑ์การนำเสนอเผยแพร่ผลงานวิจัย/วิชาการ ระดับนานาชาติ (ฉบับที่ 2 พ.ศ. 2549) ลงวันที่ 31 กรกฎาคม 2551 โดยได้แนบเอกสารที่เกี่ยวข้องมาพร้อมนี้

จึงเรียนมาเพื่อโปรดพิจารณาอนุมัติ

/s/ ฐมนวล

(ผู้ช่วยศาสตราจารย์ เจริญ ฐมนวล)

รองคณบดีฝ่ายวิจัยและบริการวิชาการ

เรียน ดร.ศุภฤกษ์ จันทร์จรัสจิตต์

เพื่อไปขอใบรับรอง

พร้อมแนบเอกสารที่เกี่ยวข้อง

๔๙-๕๐๕

29 ต.ค. 52

คณบดี

(รองศาสตราจารย์ ดร.เสกสรรค์ ด้วง)

คณบดีคณะวิศวกรรมศาสตร์

๒๘ ต.ค. ๕๒